Inferring nonlinear internal wave currents from sparse observations

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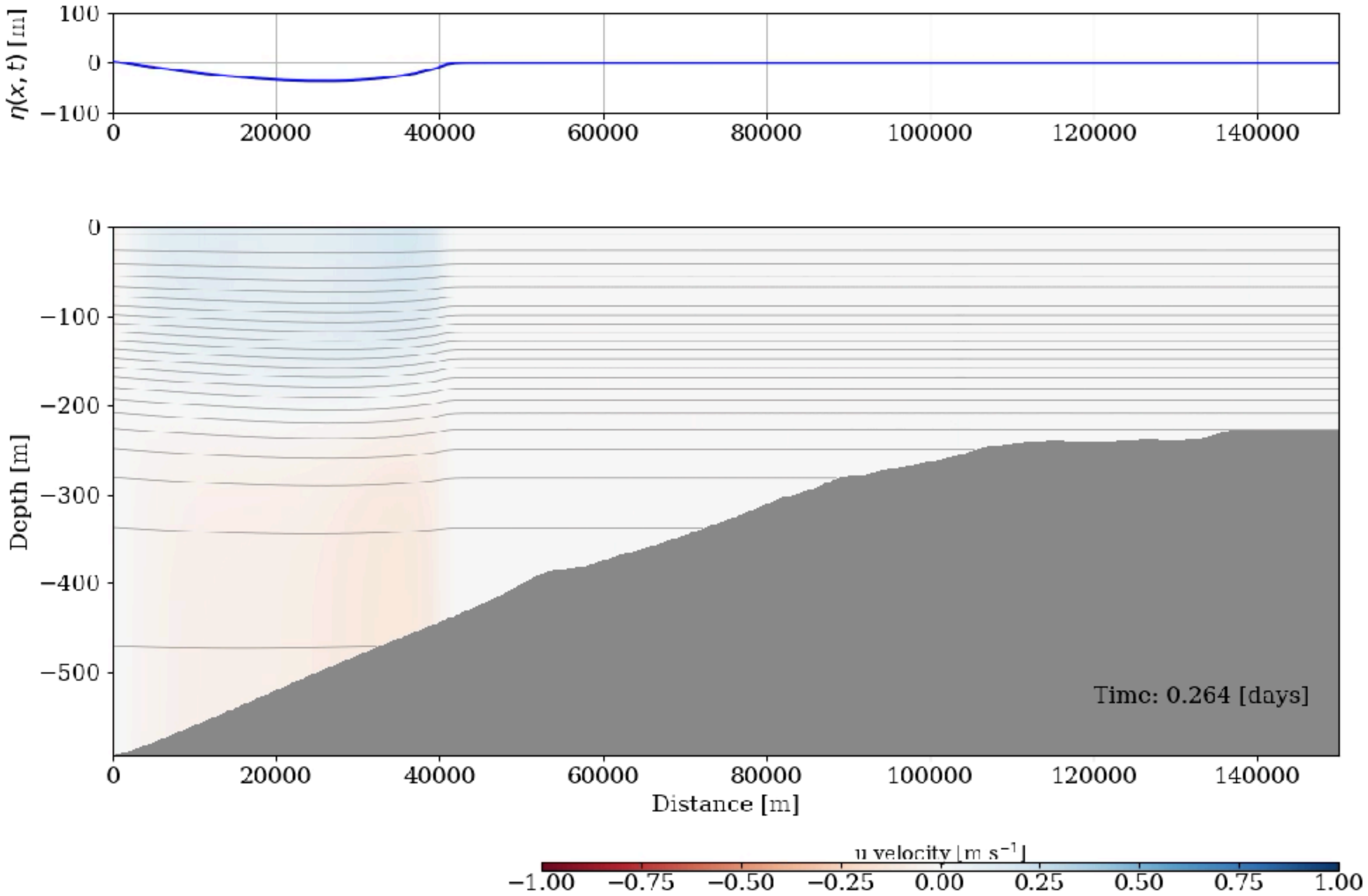
What are nonlinear internal waves…?

Nonlinear internal waves (NLIWs) are generated from the steepening of the internal tides into large aperiodic internal waves.



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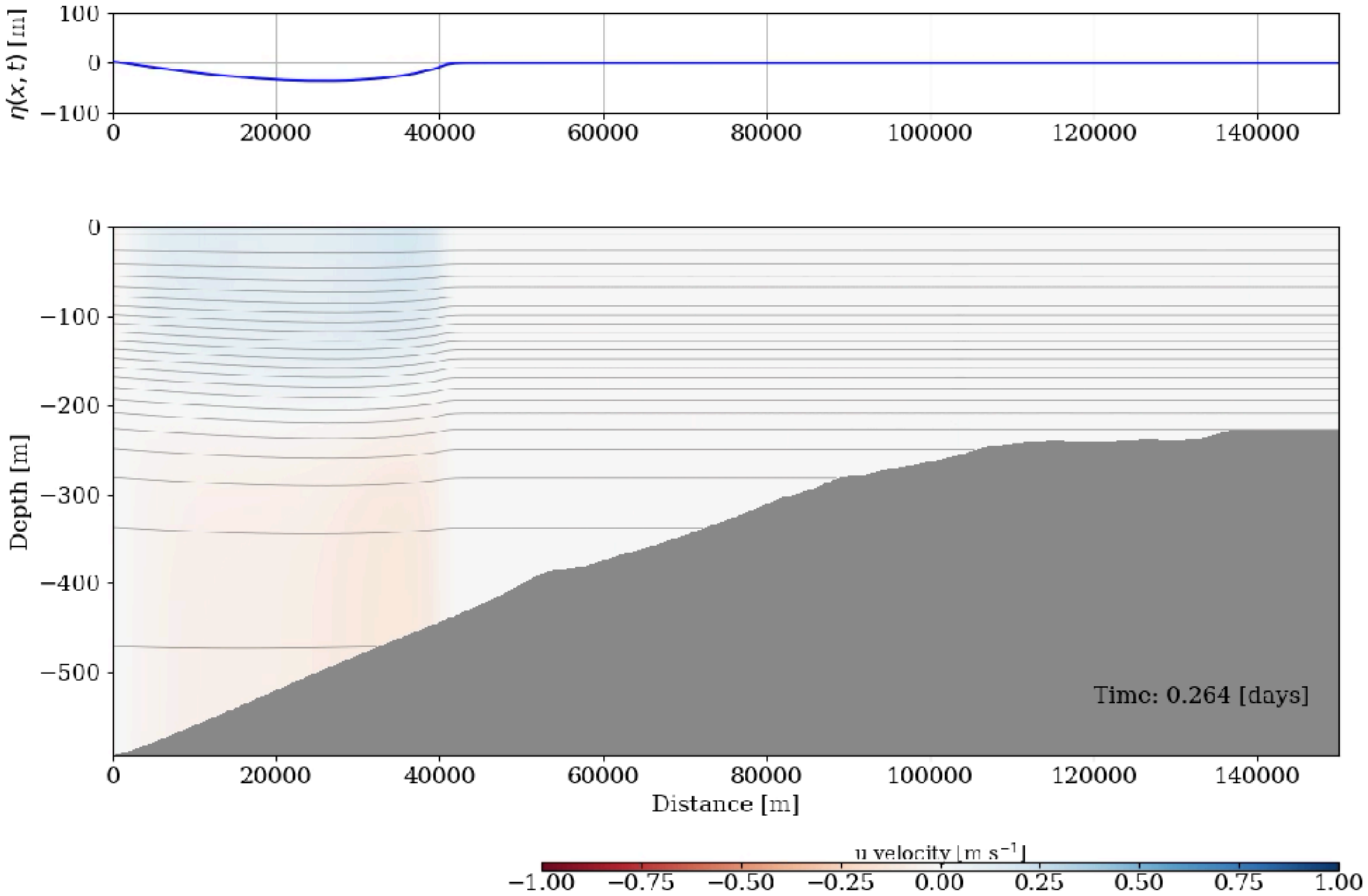
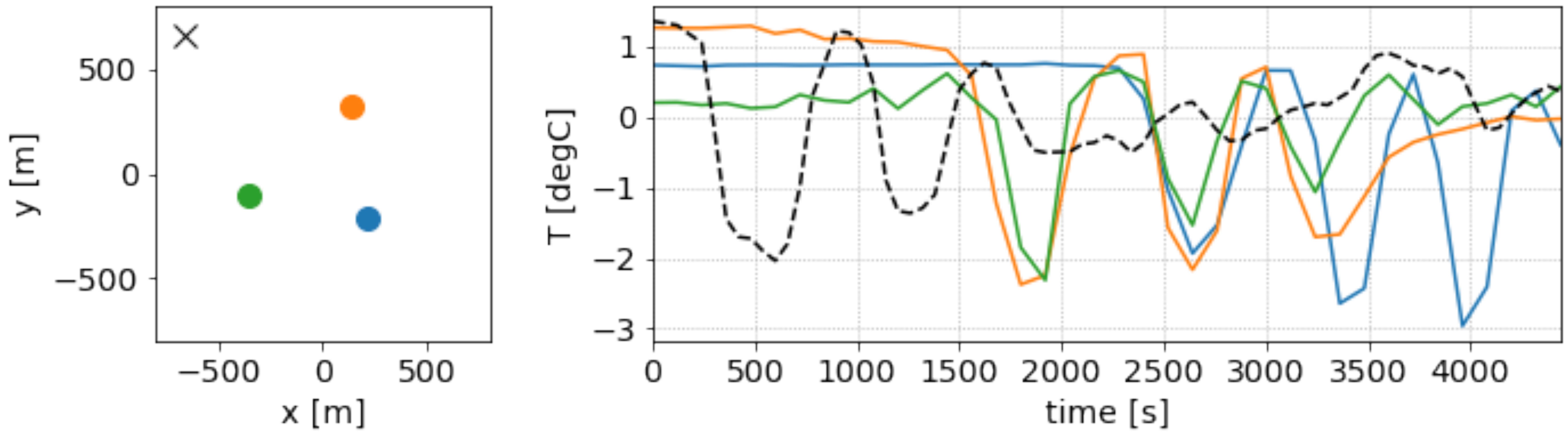


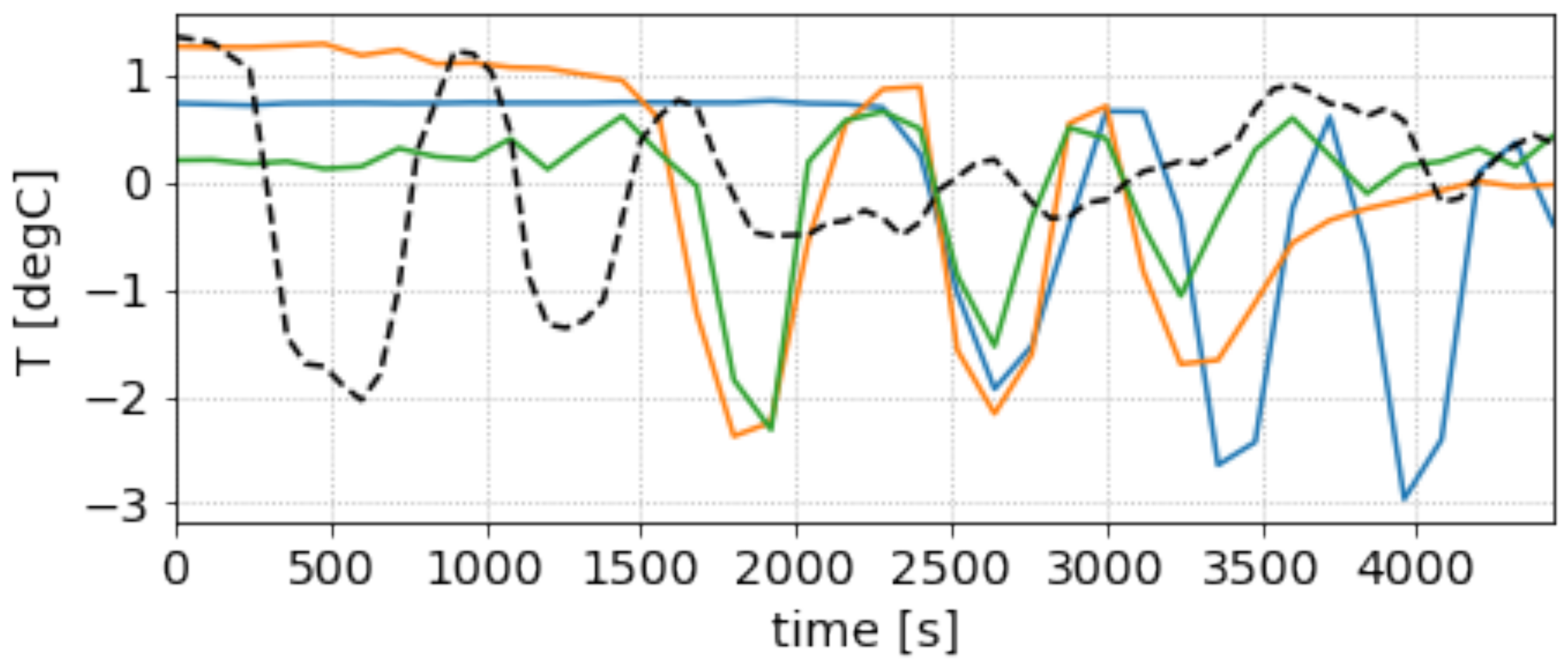
…and why do we care?

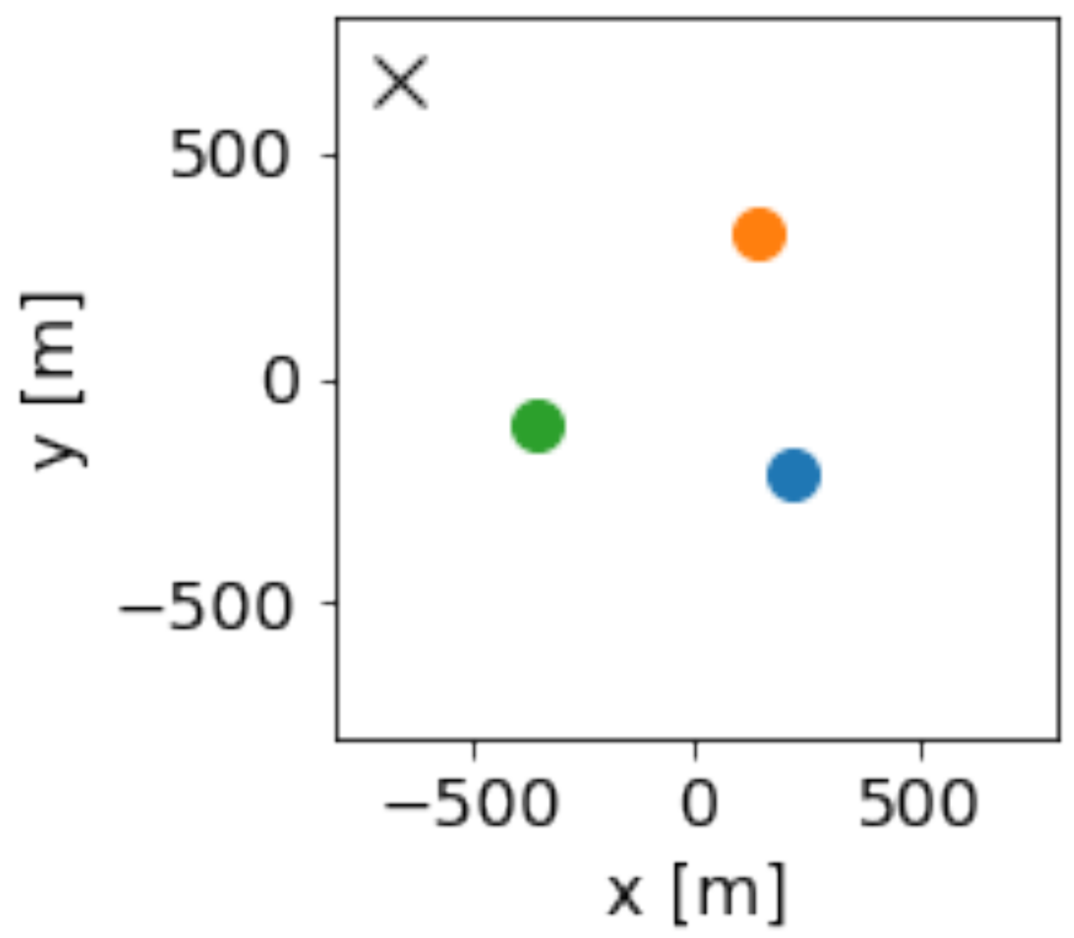
NLIWs generate extreme currents which dictate design criteria for many structures on the continental shelf.



How, historically, are internal waves modelled?

- Numerical modelling (3D non-hydrostatic Reynolds-averaged ocean models)
  - Too computationally expensive
- Regression (cnoidal wave functions)
  - Functions too restrictive
- Machine learning (neural networks)
  - No physical interpretability





Synthesising physics and statistical learning

- A Gaussian process is an infinite dimensional prior distribution over functions

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- We parameterise NLIW (and potentially other) structure into these functions

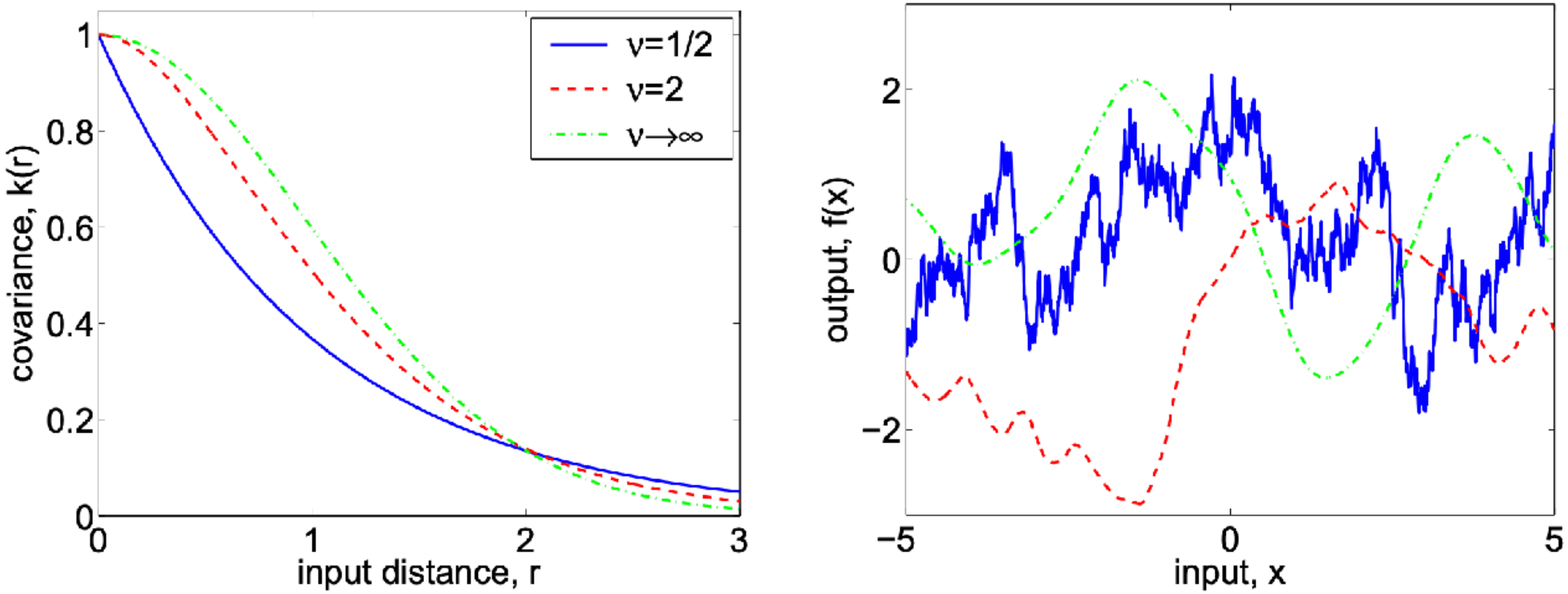
Synthesising physics and statistical learning

- ~~A Gaussian process is an infinite dimensional prior distribution over functions~~
- A Gaussian process is a big multivariate normal distribution
- Spatio-temporal structure is captured by the mean and covariance functions
- We parameterise NLIW (and potentially other) structure into these functions
- Similar to Kriging and Optimal Interpolation, BUT, offers a robust way to infer parameters

Embedding structure into the covariance

A covariance function, $k(\cdot, \cdot)$, describes the covariance between two locations.

A Matérn covariance function with different parameterisations:



Rasmussen and Williams (2006)

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Stationarity (covariance a function of distance in input space):

$$k(\mathbf{x}, \mathbf{x}') = k(|\mathbf{x} - \mathbf{x}'|, 0) = k(\tau), \text{ where } \tau = |\mathbf{x} - \mathbf{x}'|$$

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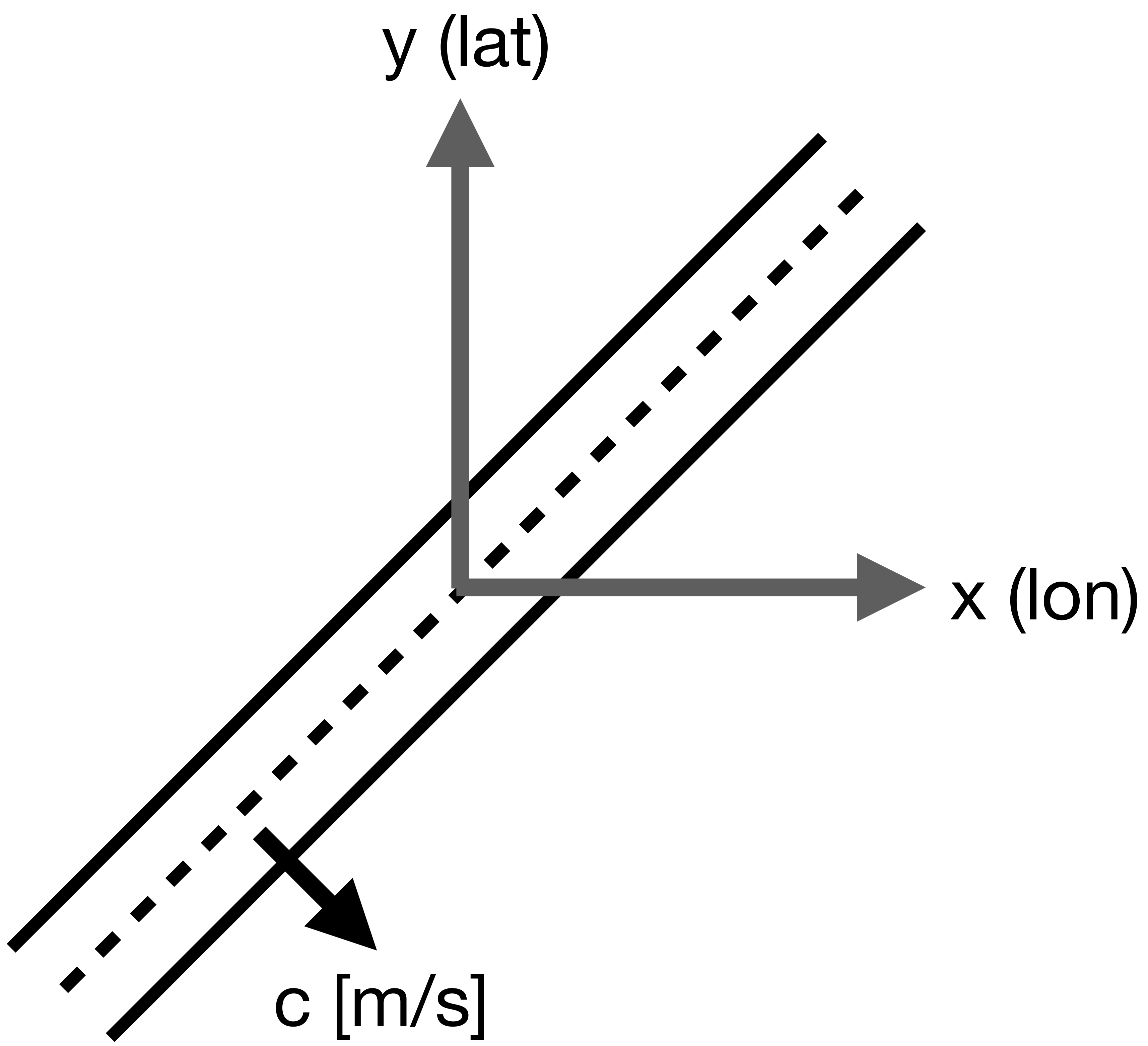
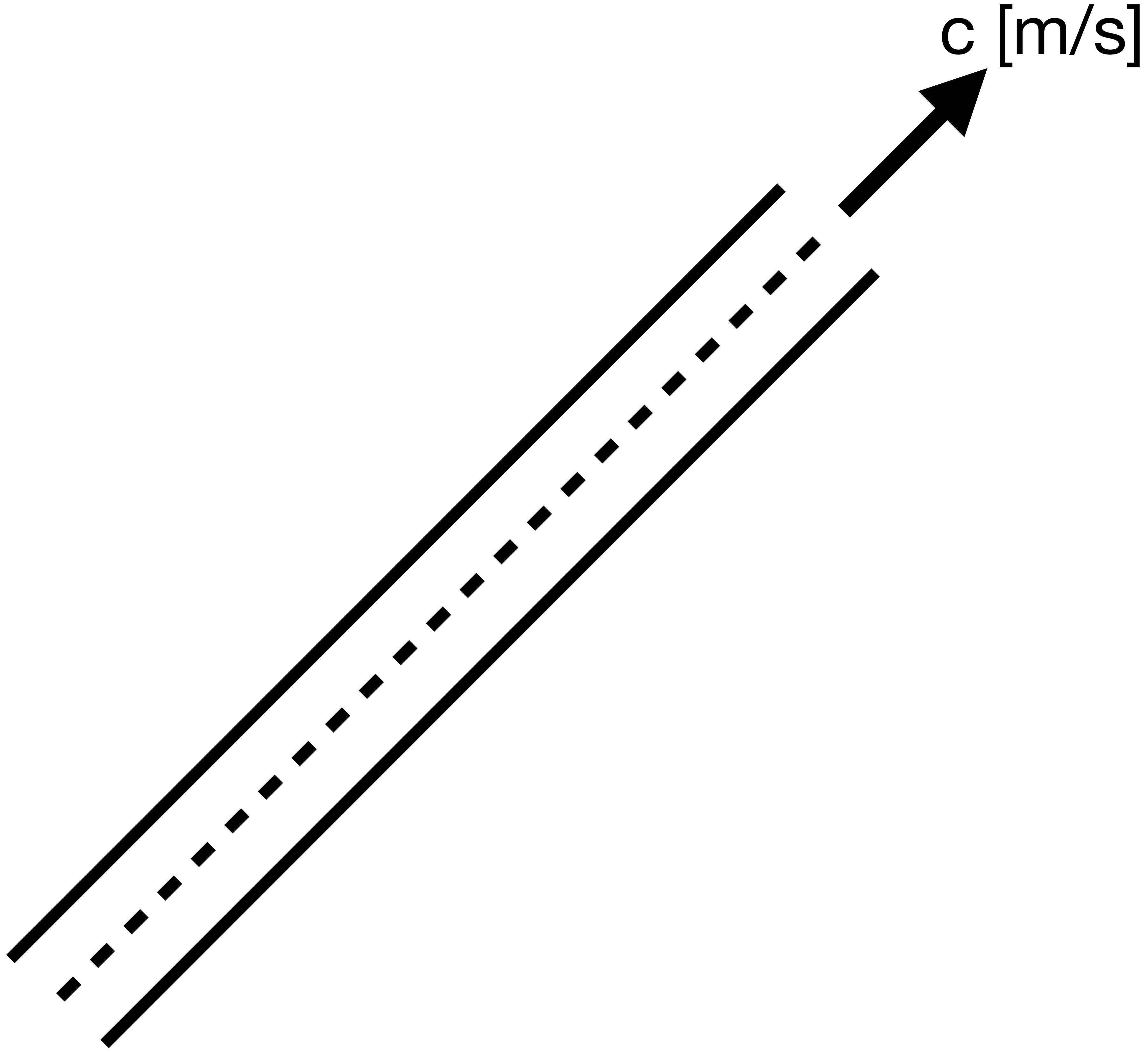
Anisotropy (covariance in each dimension is different):

$$k_i(\cdot\,, \cdot) \neq k(\cdot\,, \cdot), \text{ for } i \in \{x, y, t\}$$

Embedding *NLIW dynamics* into the covariance



Latitude/Longitude Projection



Across-crest/Time Projection

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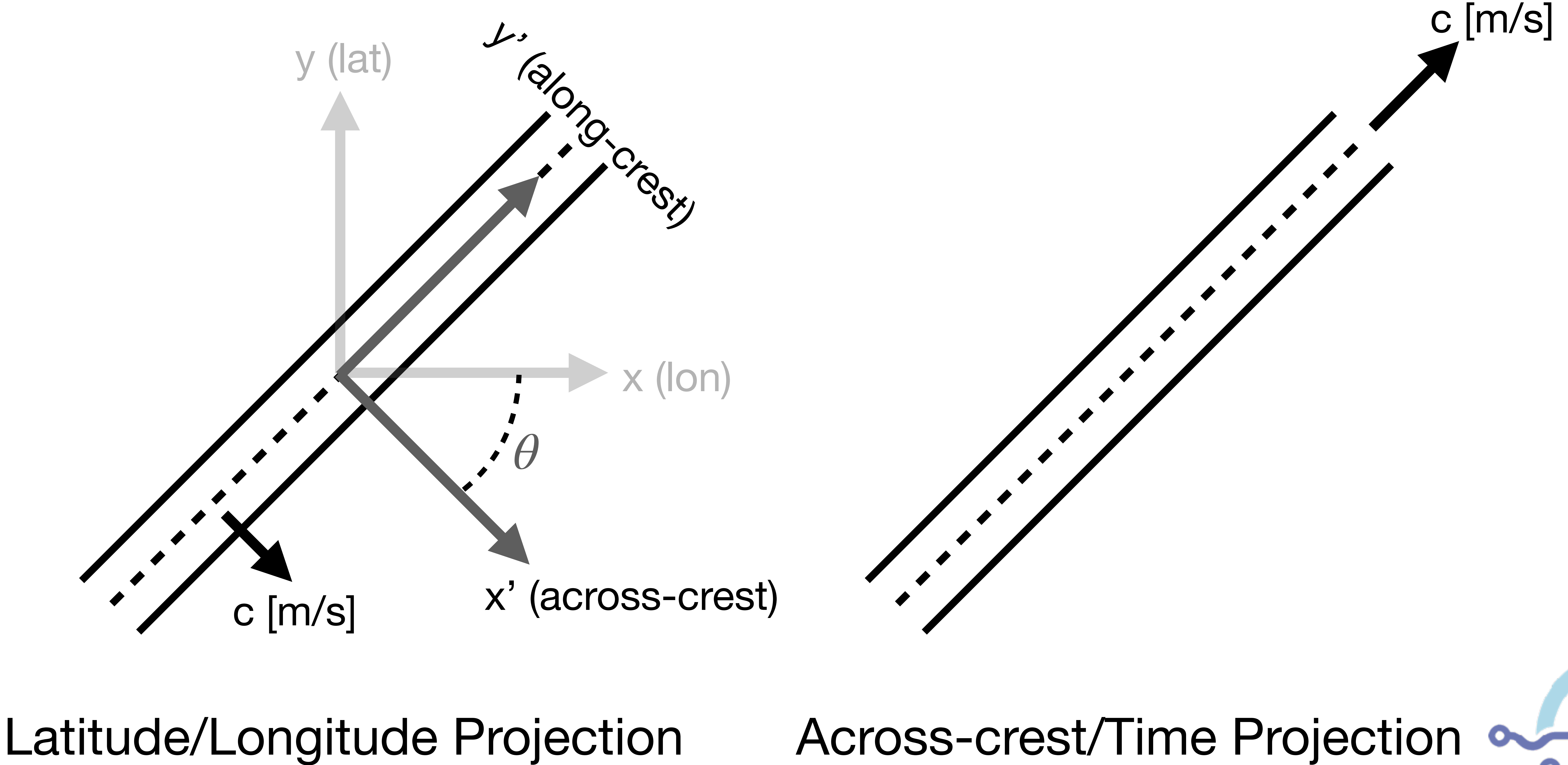

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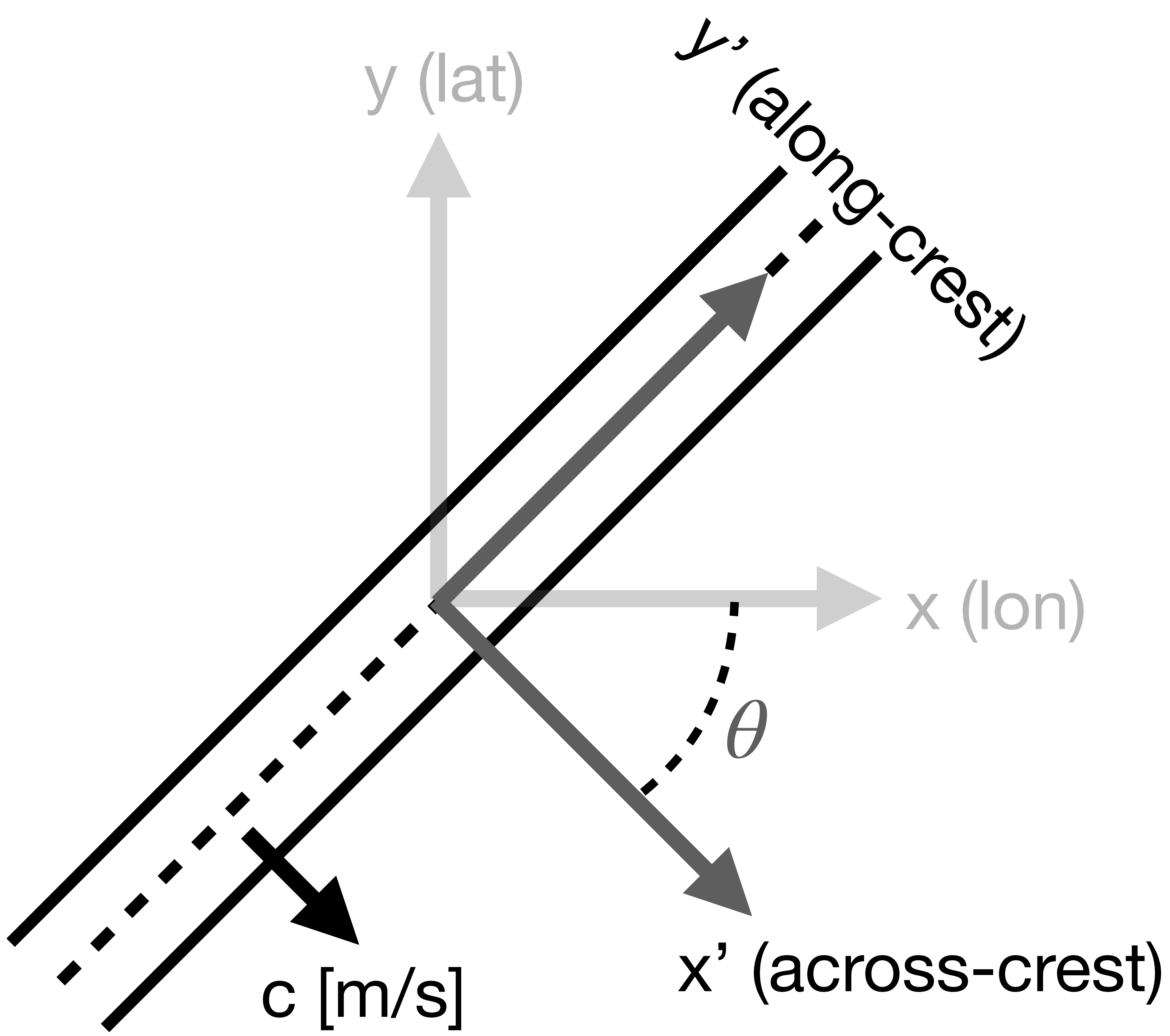

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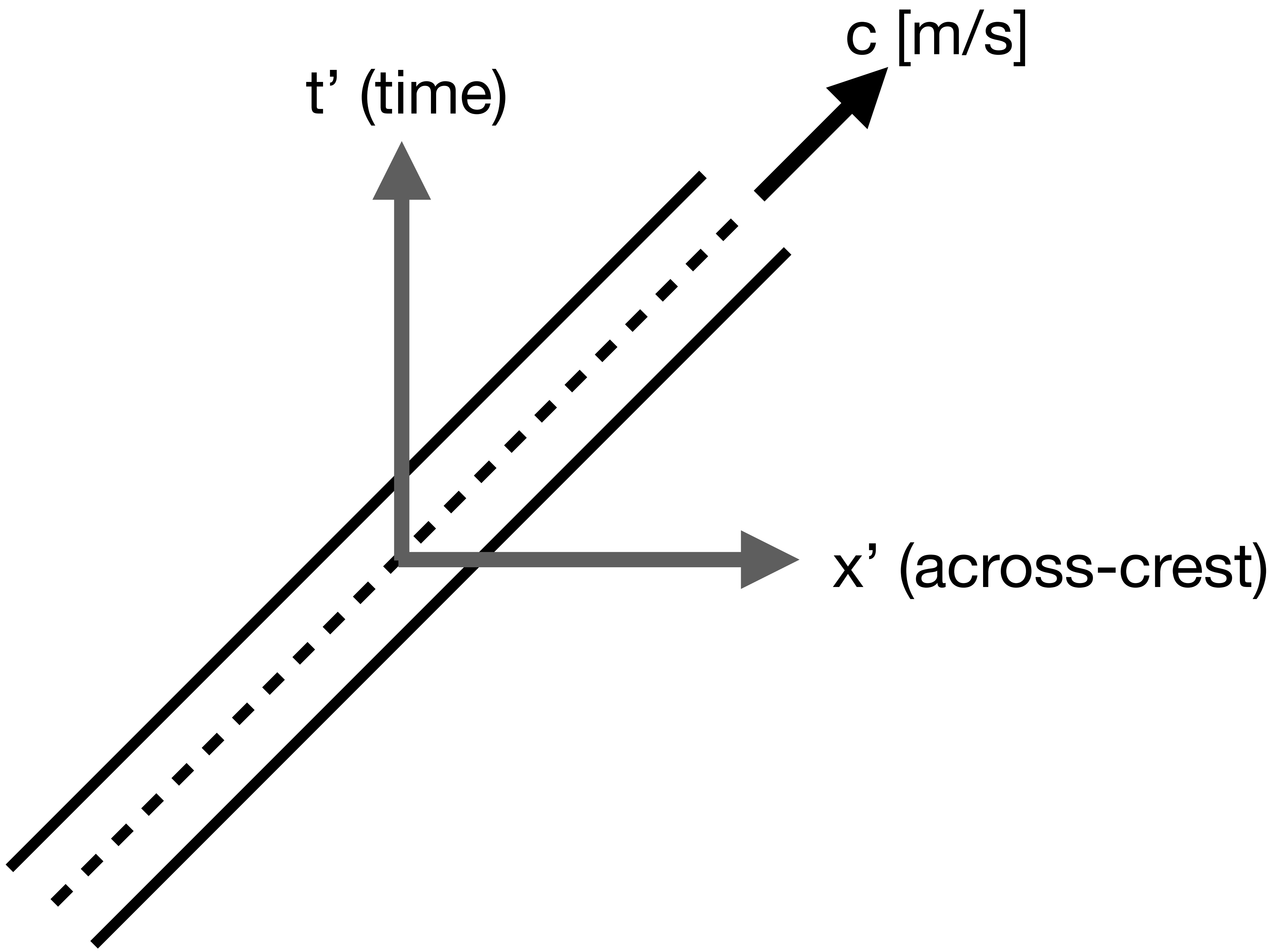

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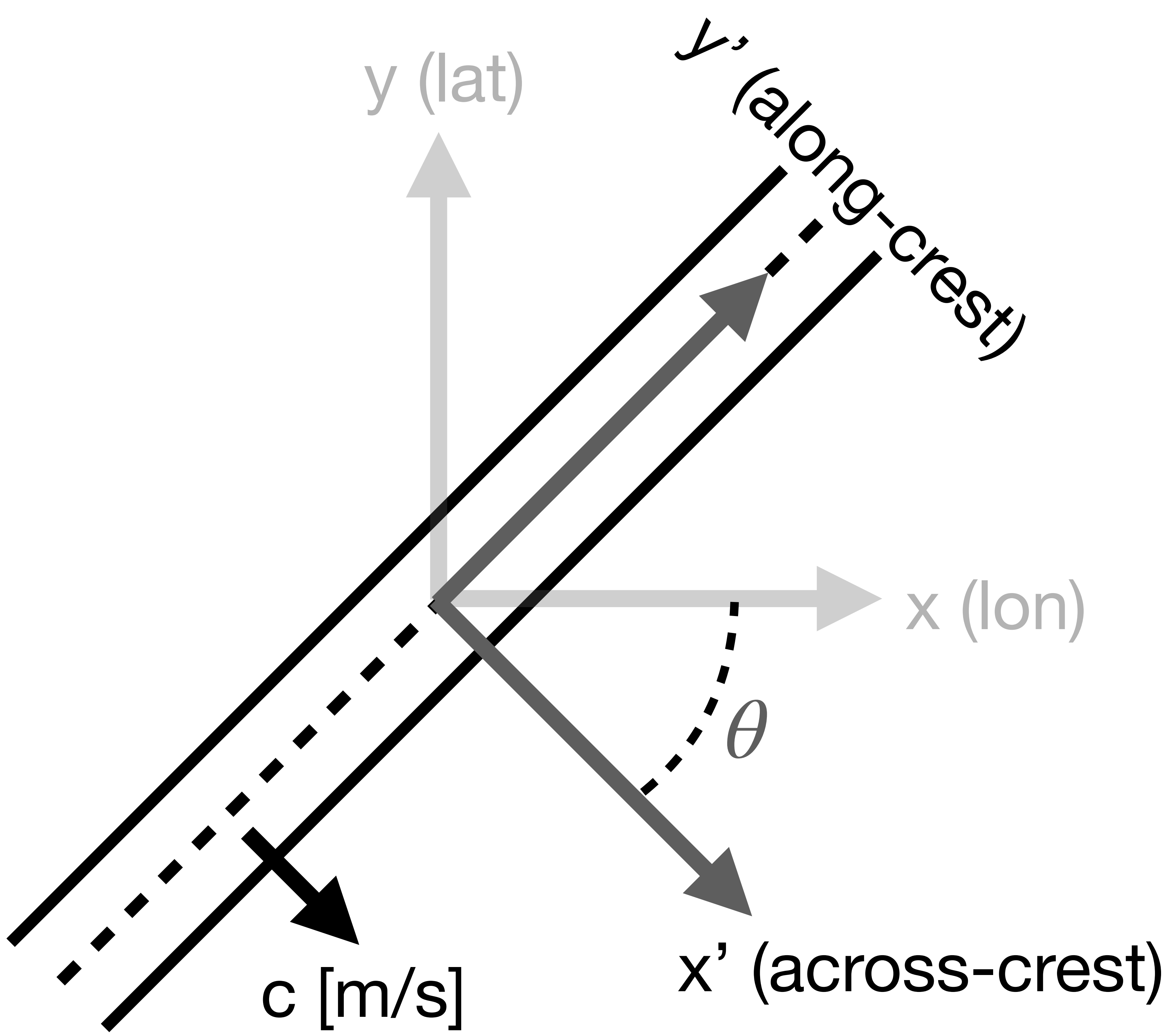

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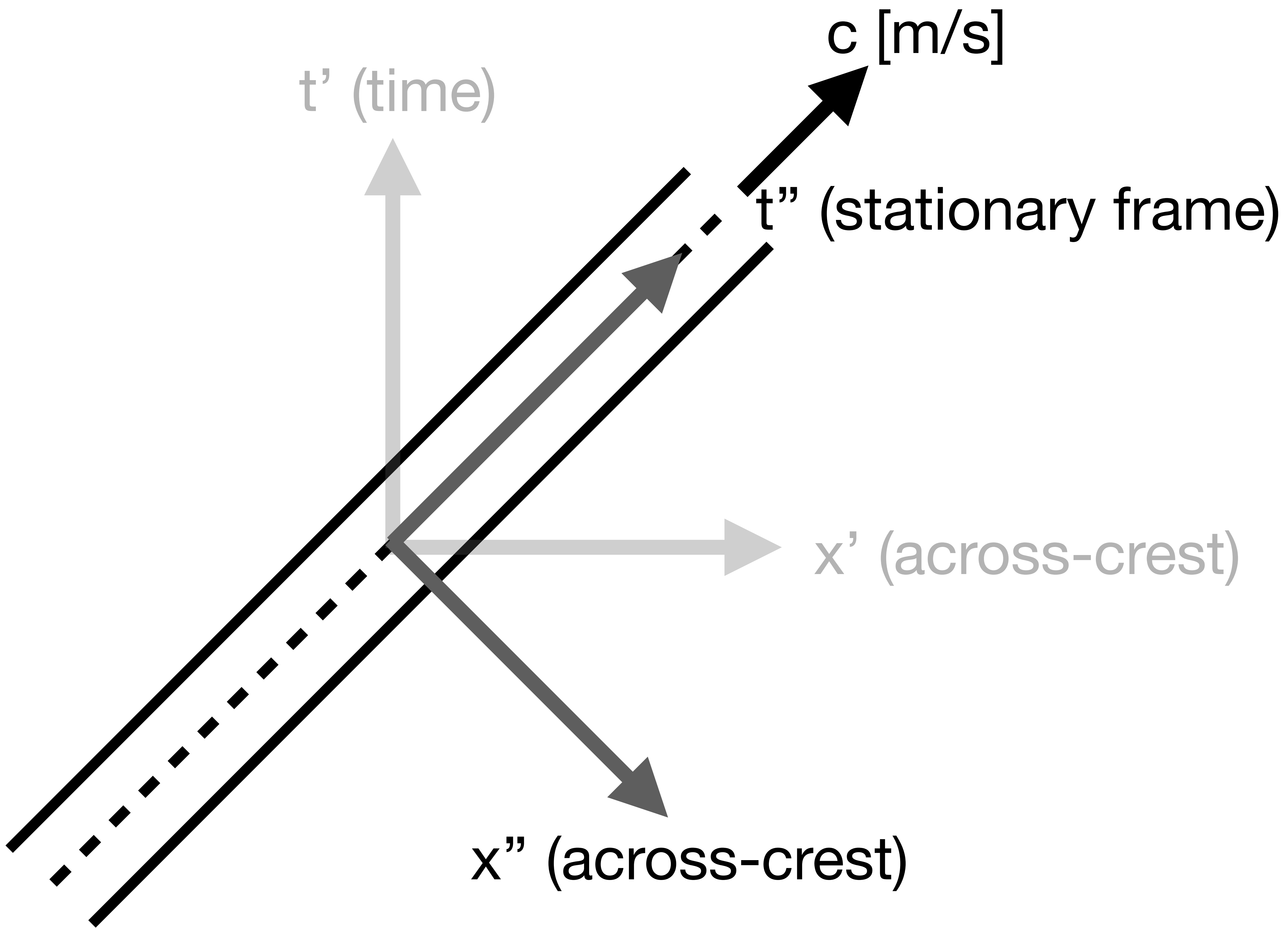

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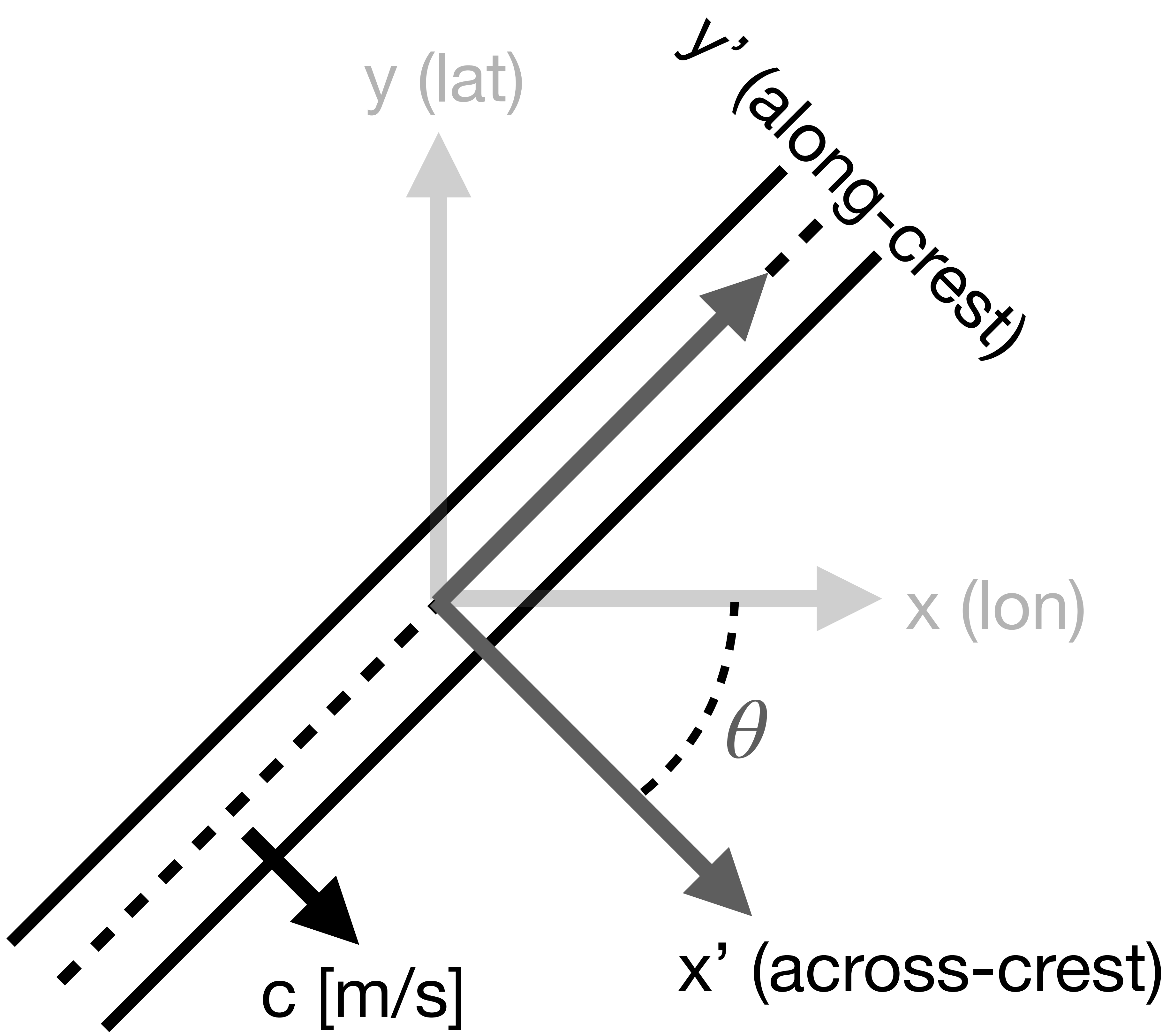

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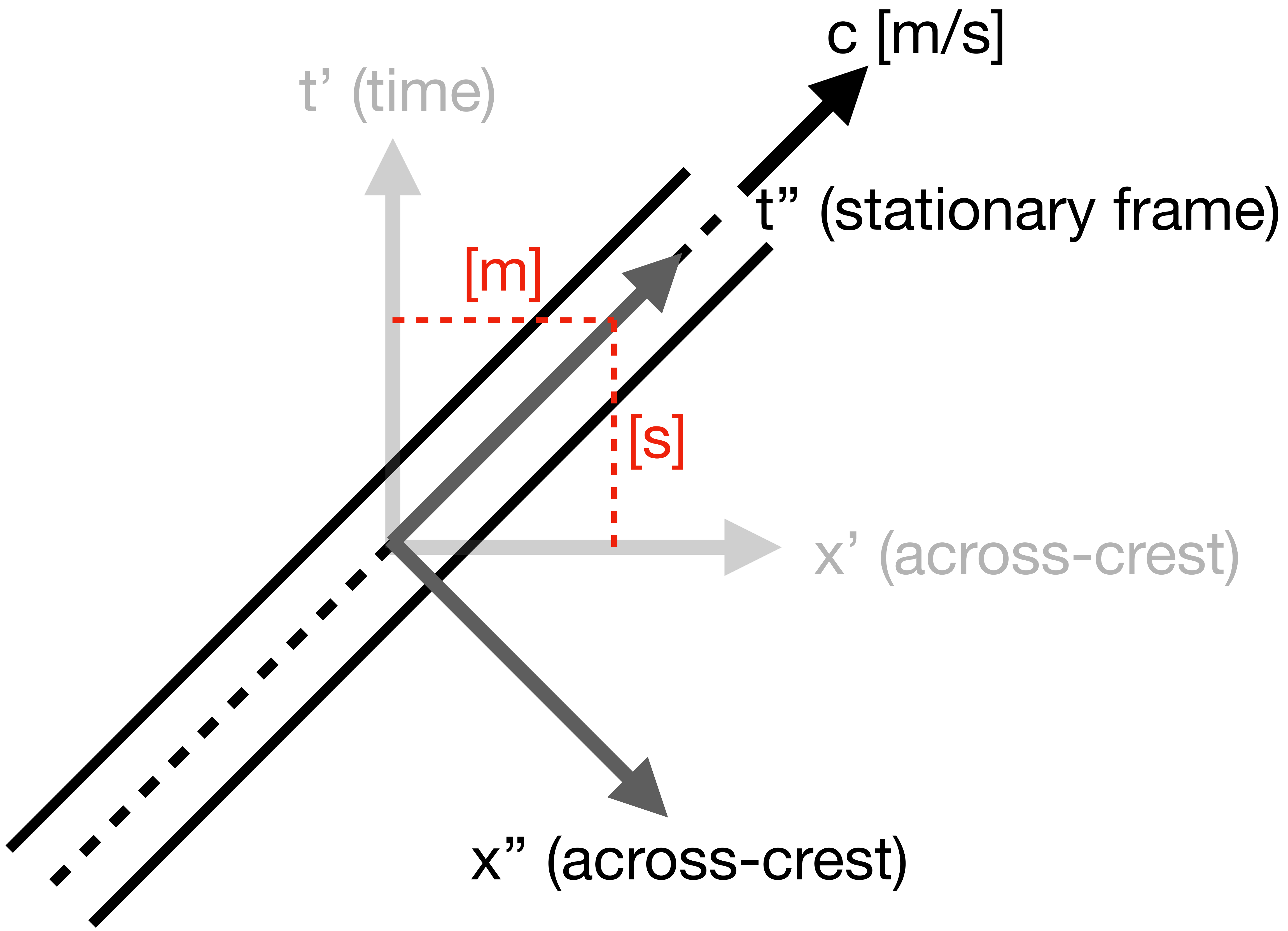

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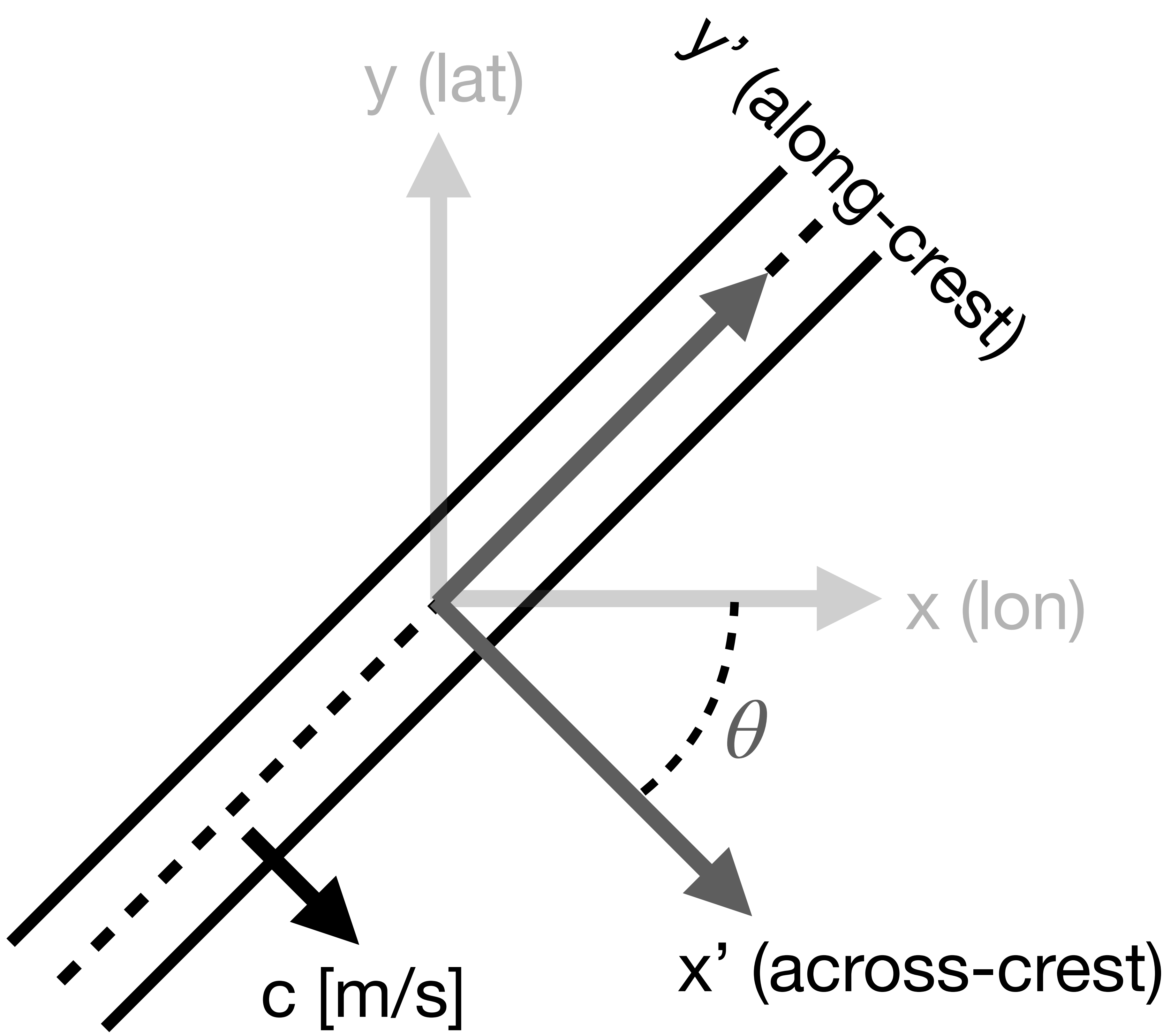

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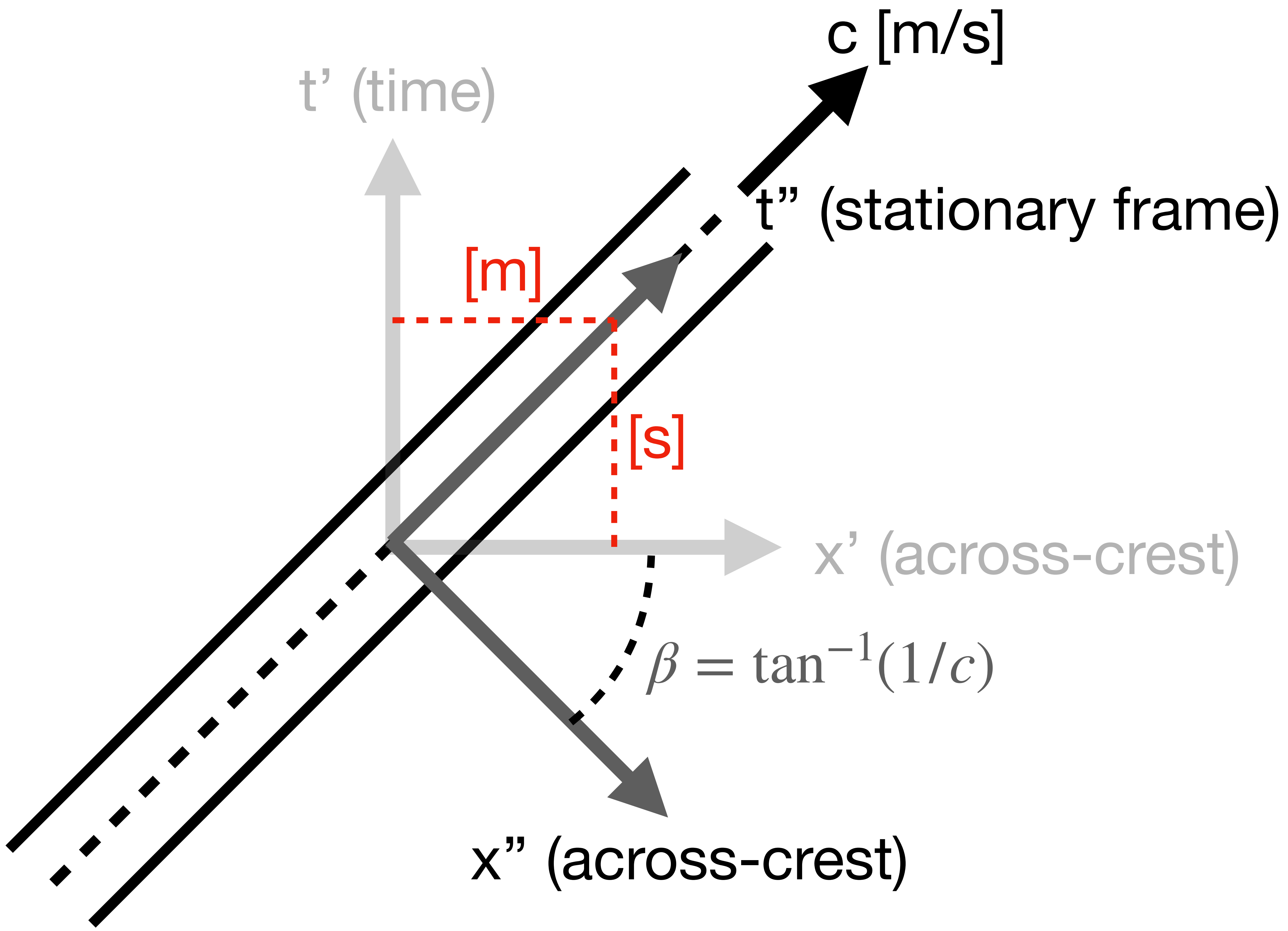

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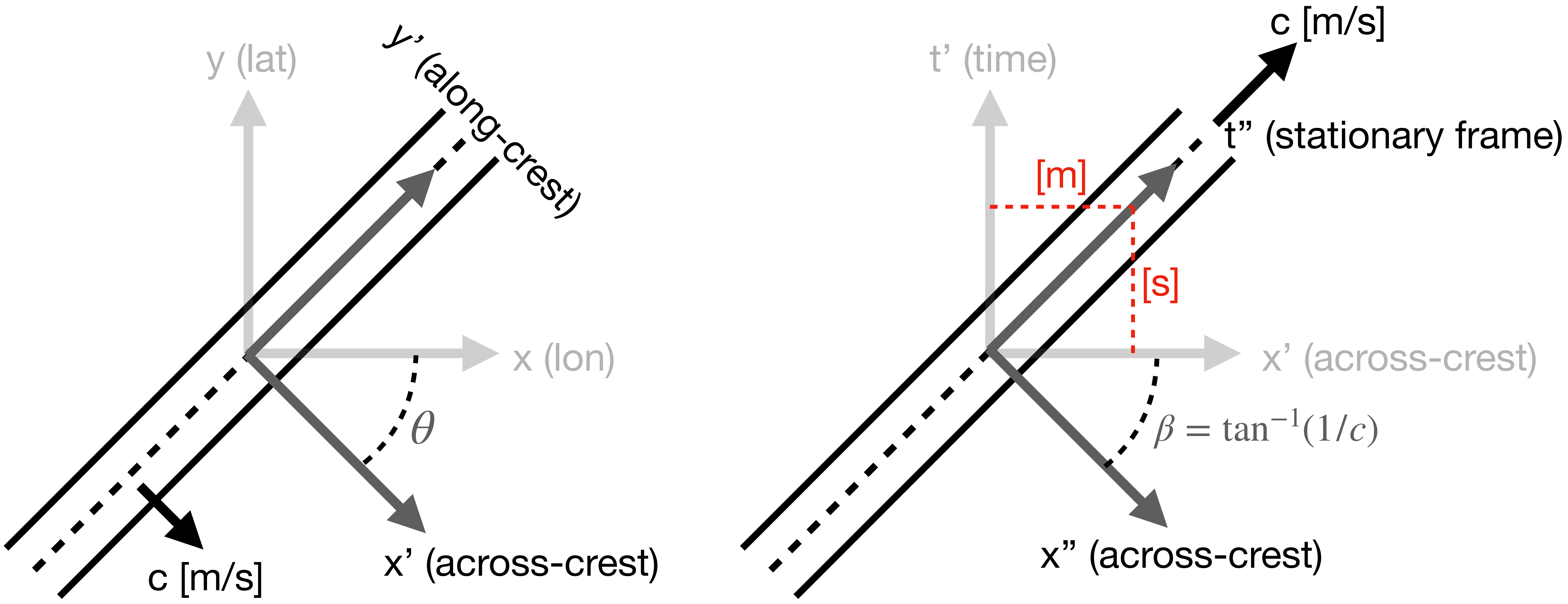
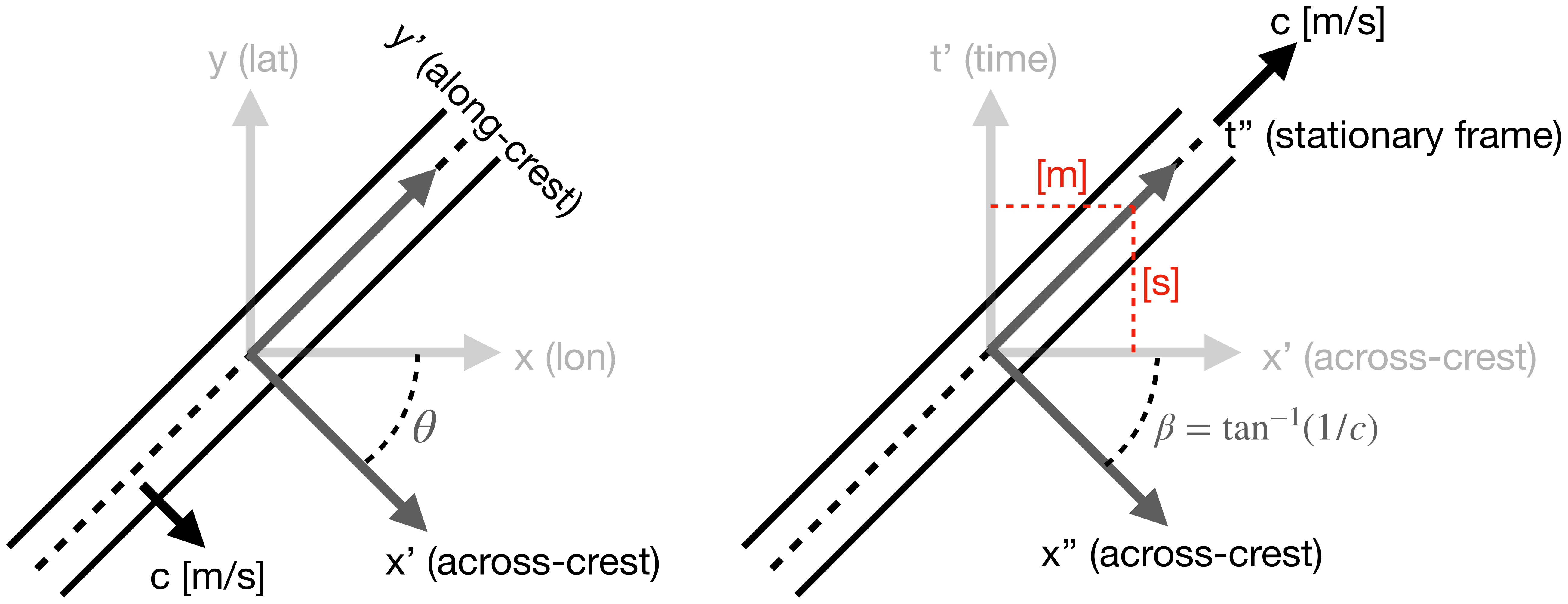


Embedding *NLIW dynamics* into the covariance

$$k_{\mathrm{NLIW}}(\tau;\theta) = \sigma^2\, k_{x''}(\tau_{x''};\theta)\, k_{y''}(\tau_{y''};\theta)\, k_{t''}(\tau_{t''};\theta)$$



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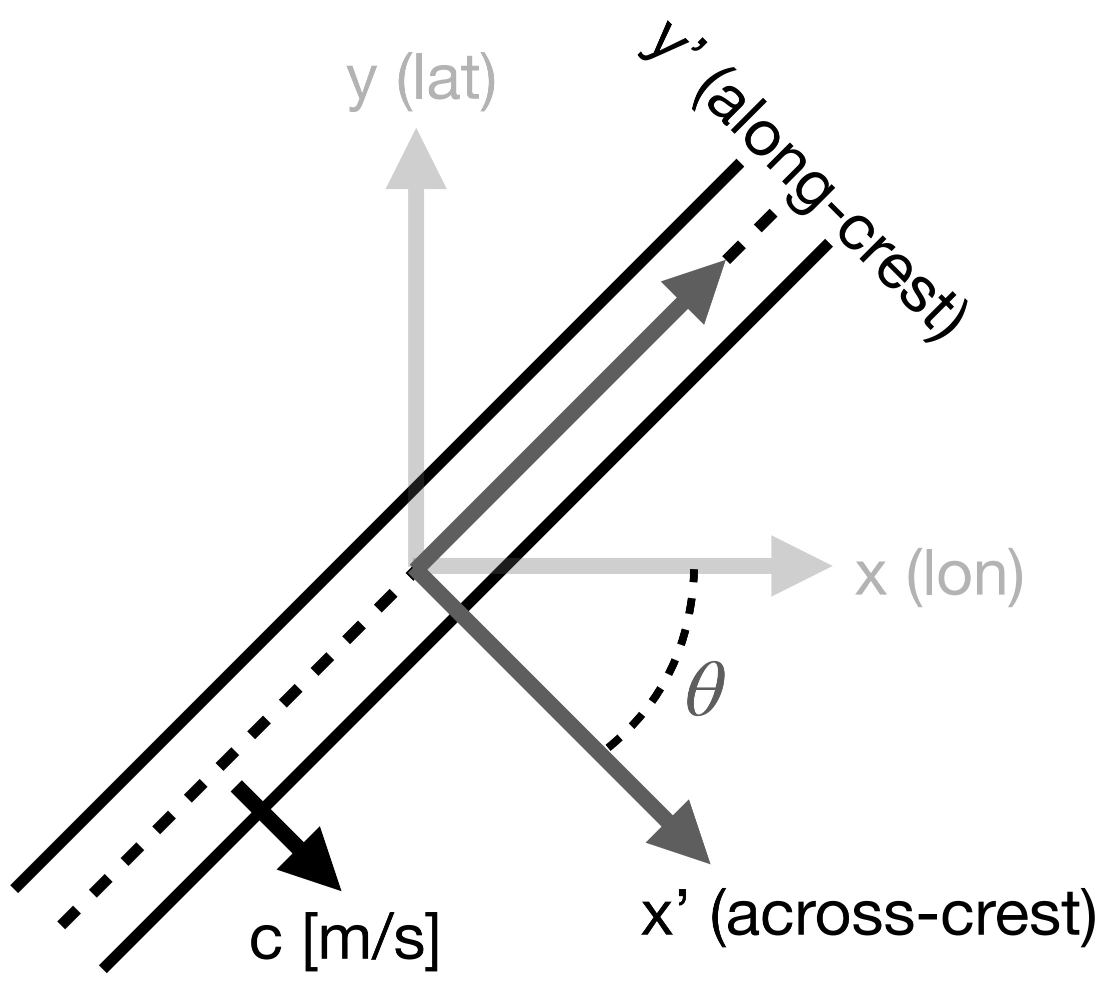

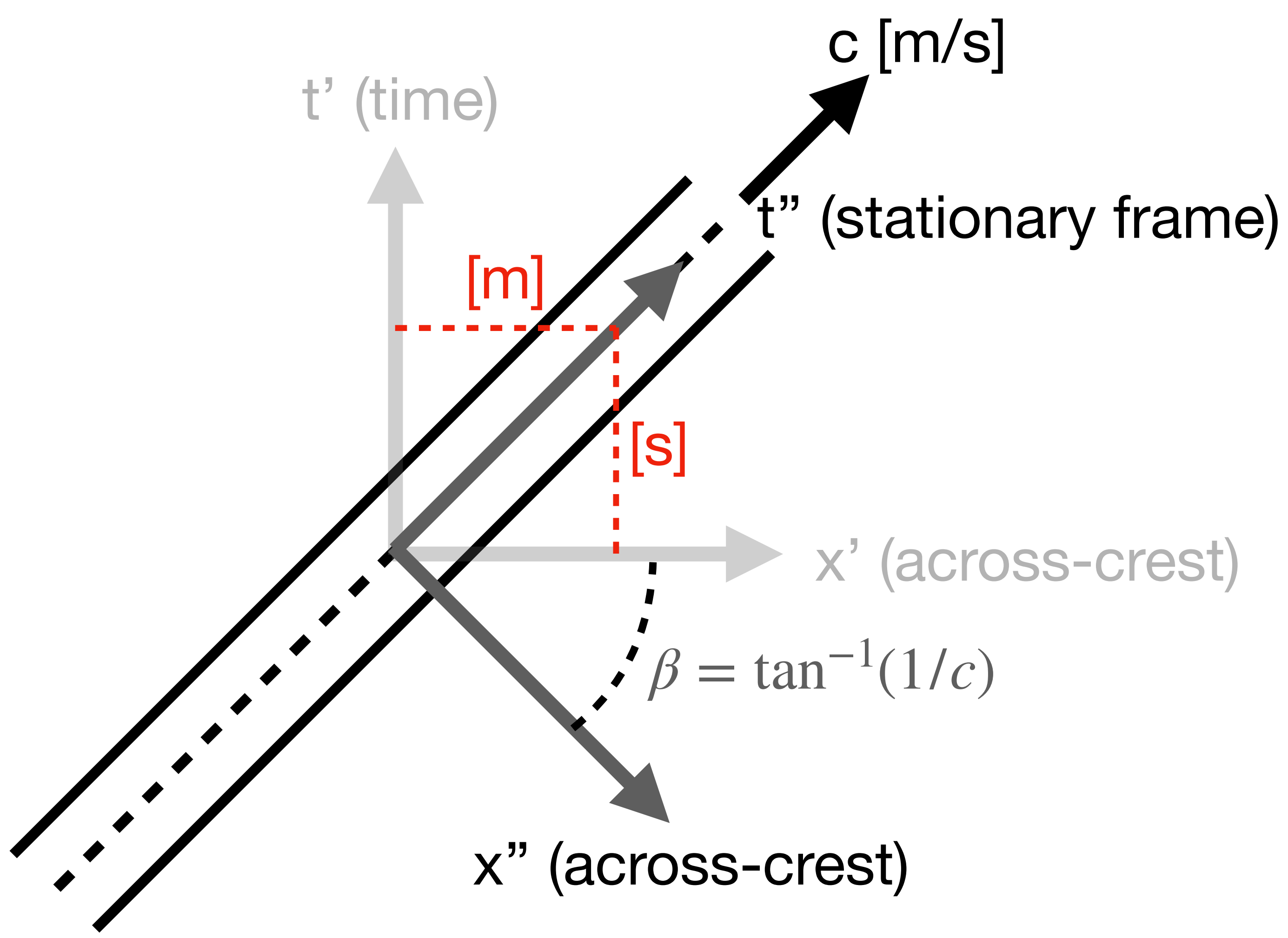

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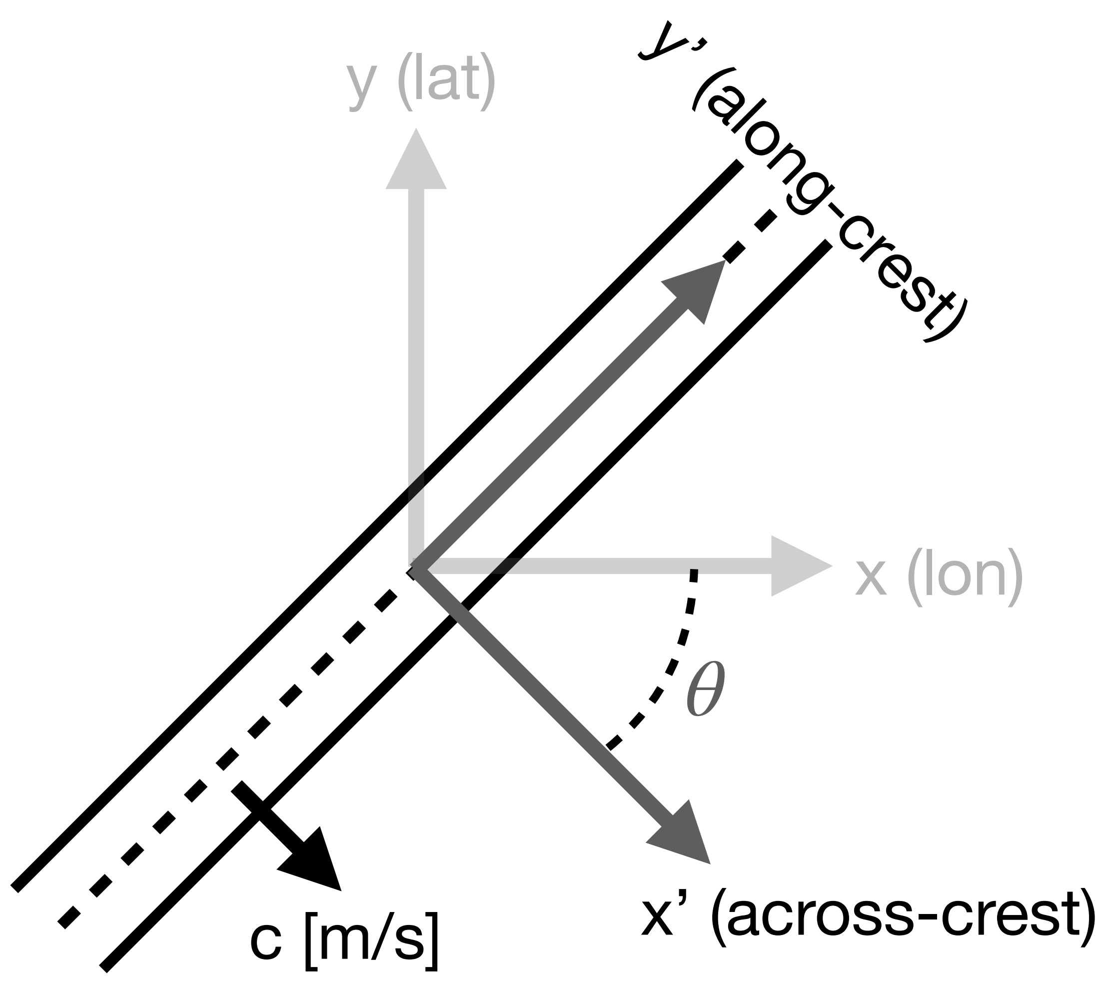
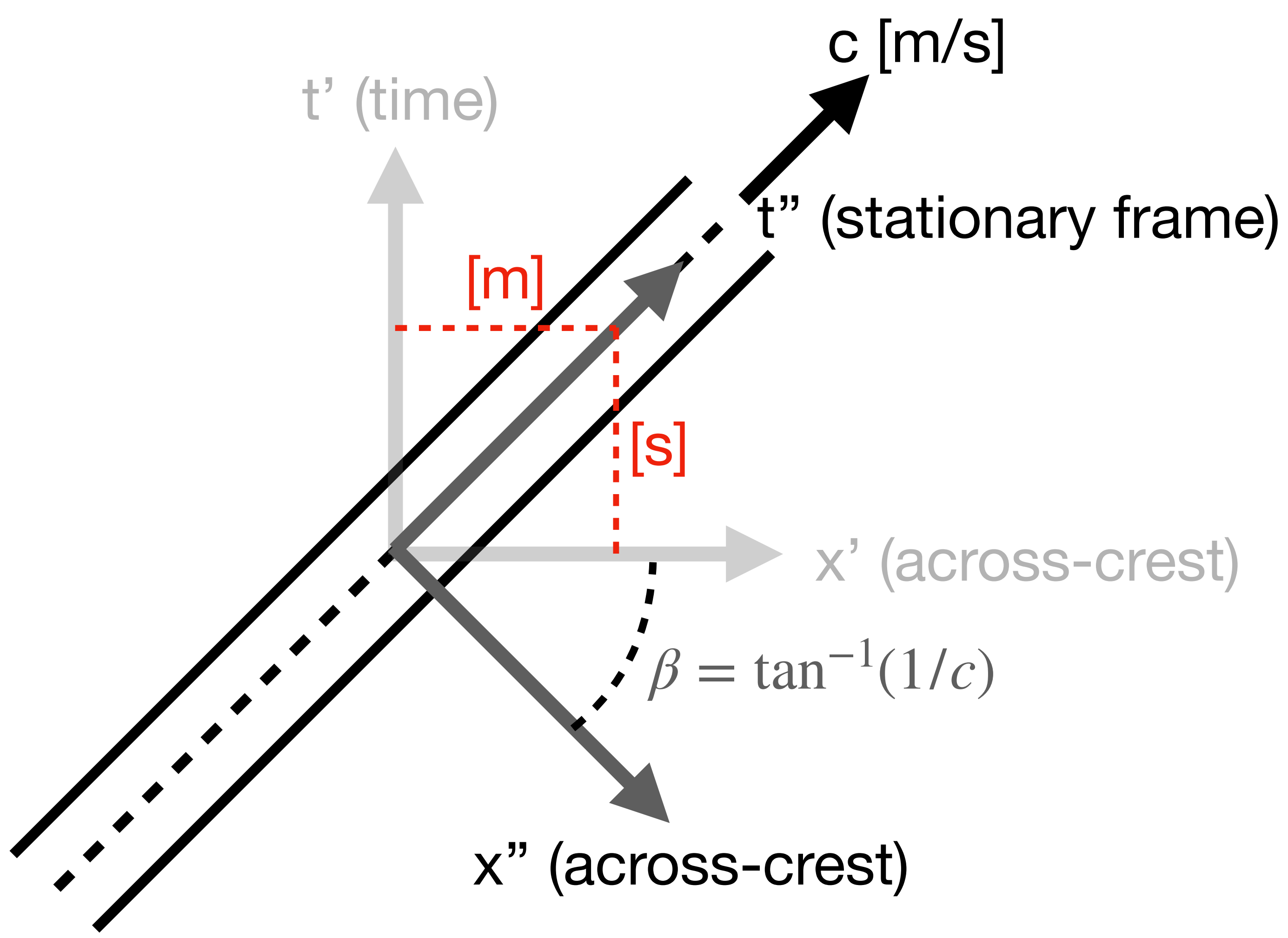




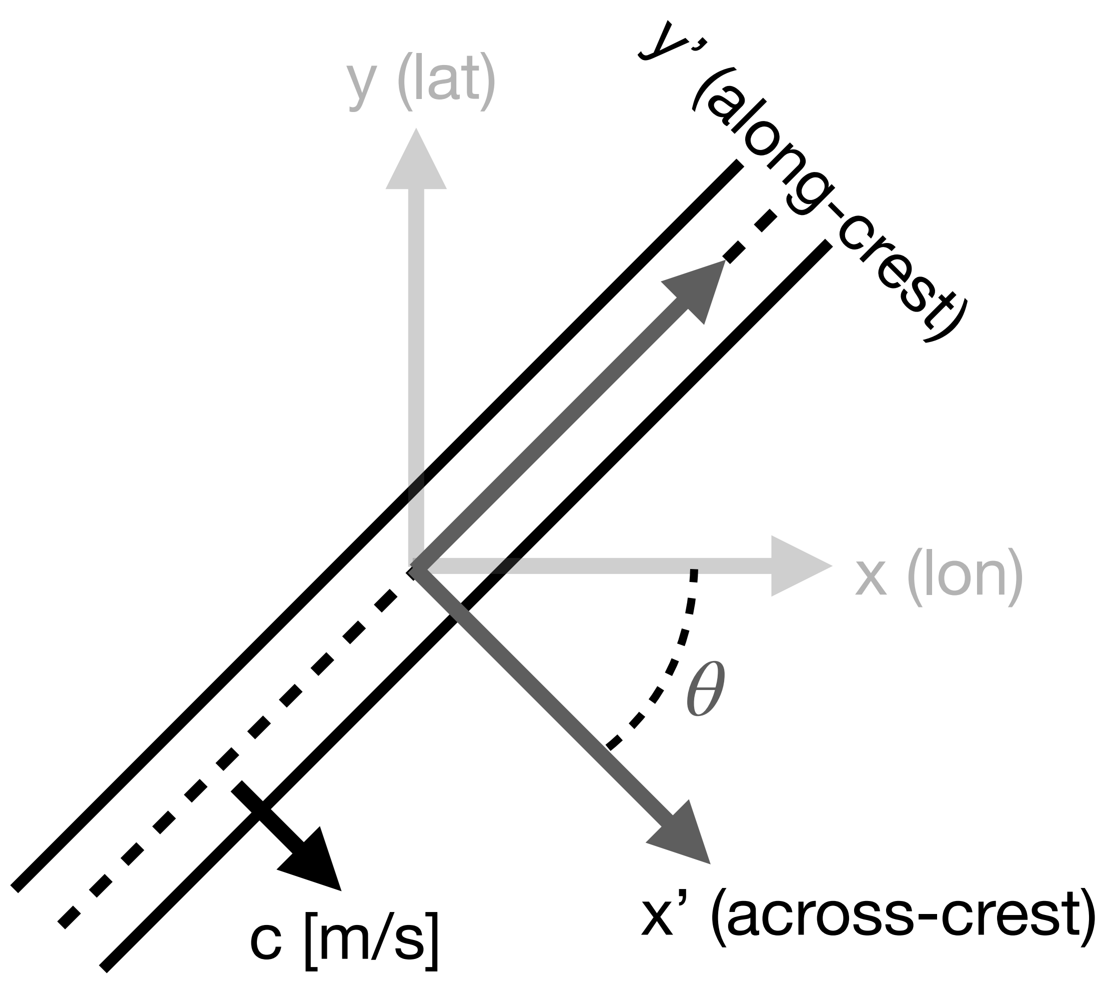
Embedding *NLIW dynamics* into the covariance

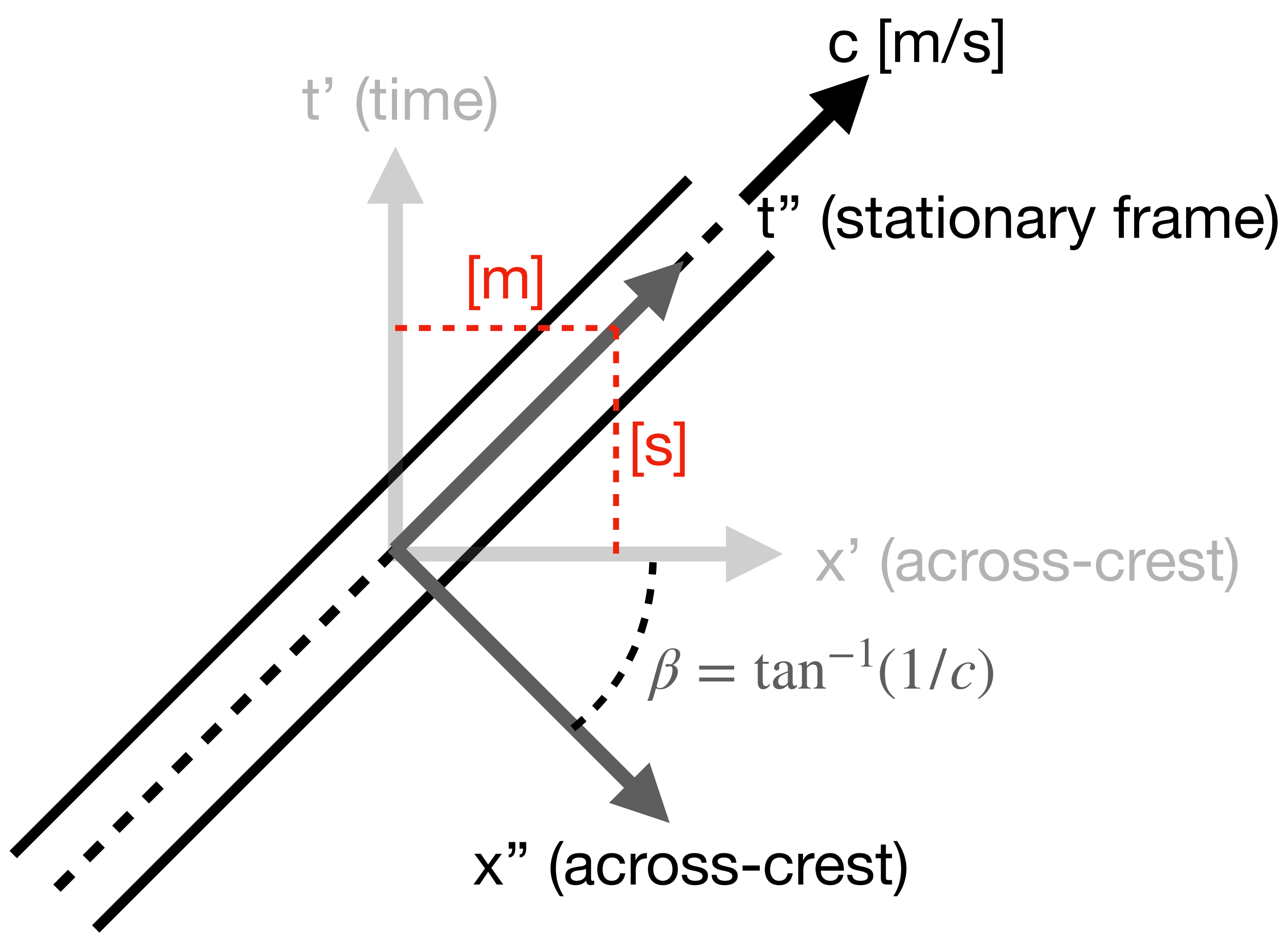
Across-crest

Decay Term

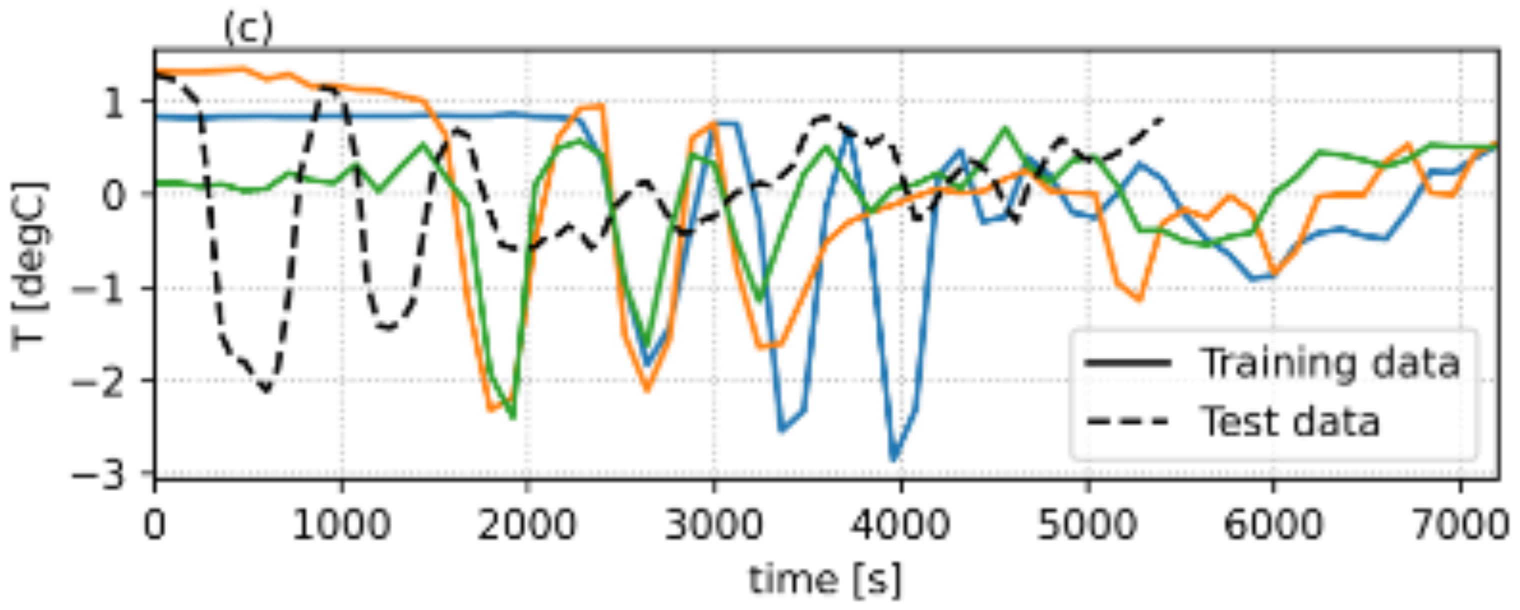
$$k_{\mathrm{NLIW}}(\tau;\theta) = \sigma^2\, k_{x''}(\tau_{x''};\theta)\, k_{y''}(\tau_{y''};\theta)\, k_{t''}(\tau_{t''};\theta)$$

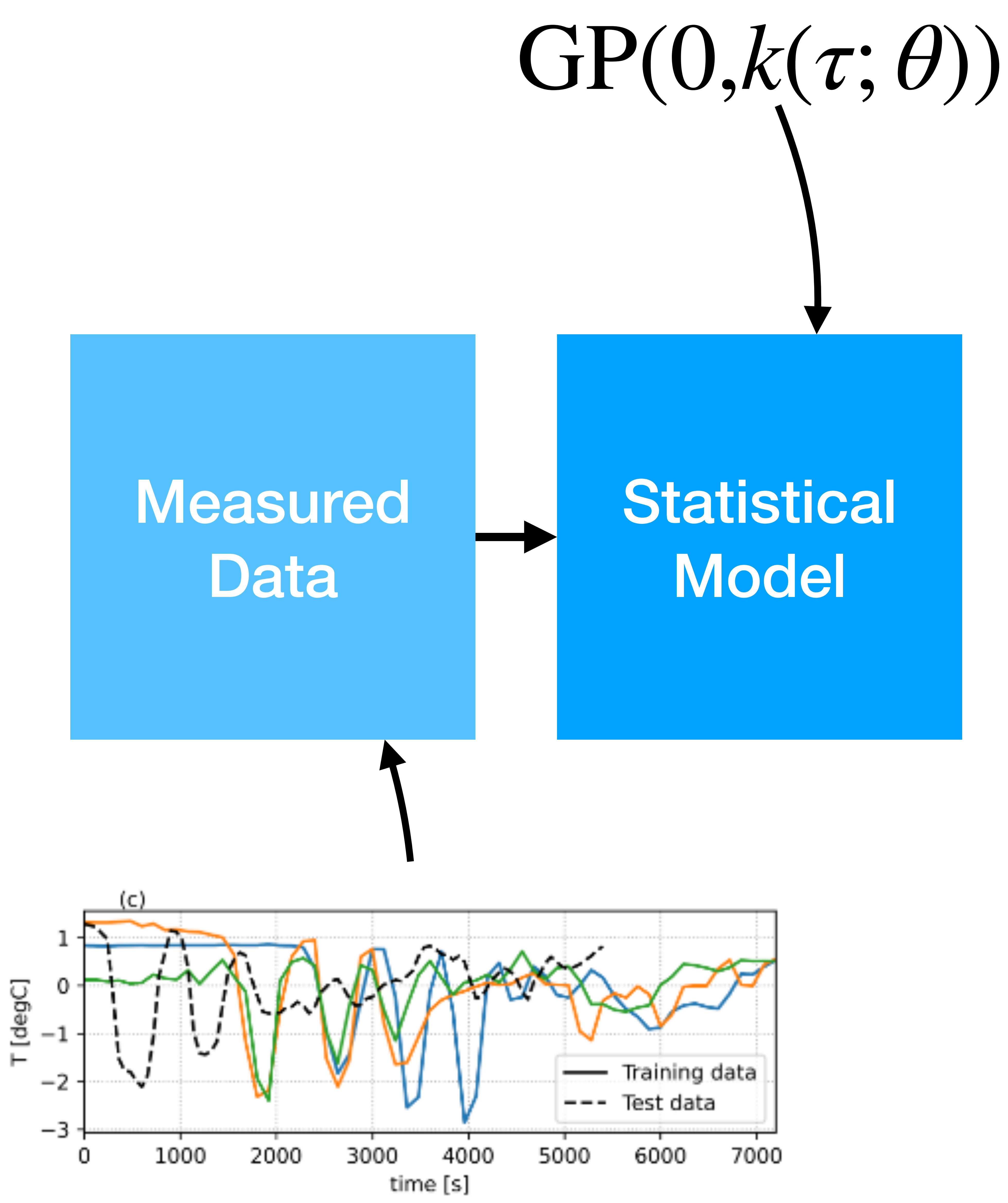
Along-crest



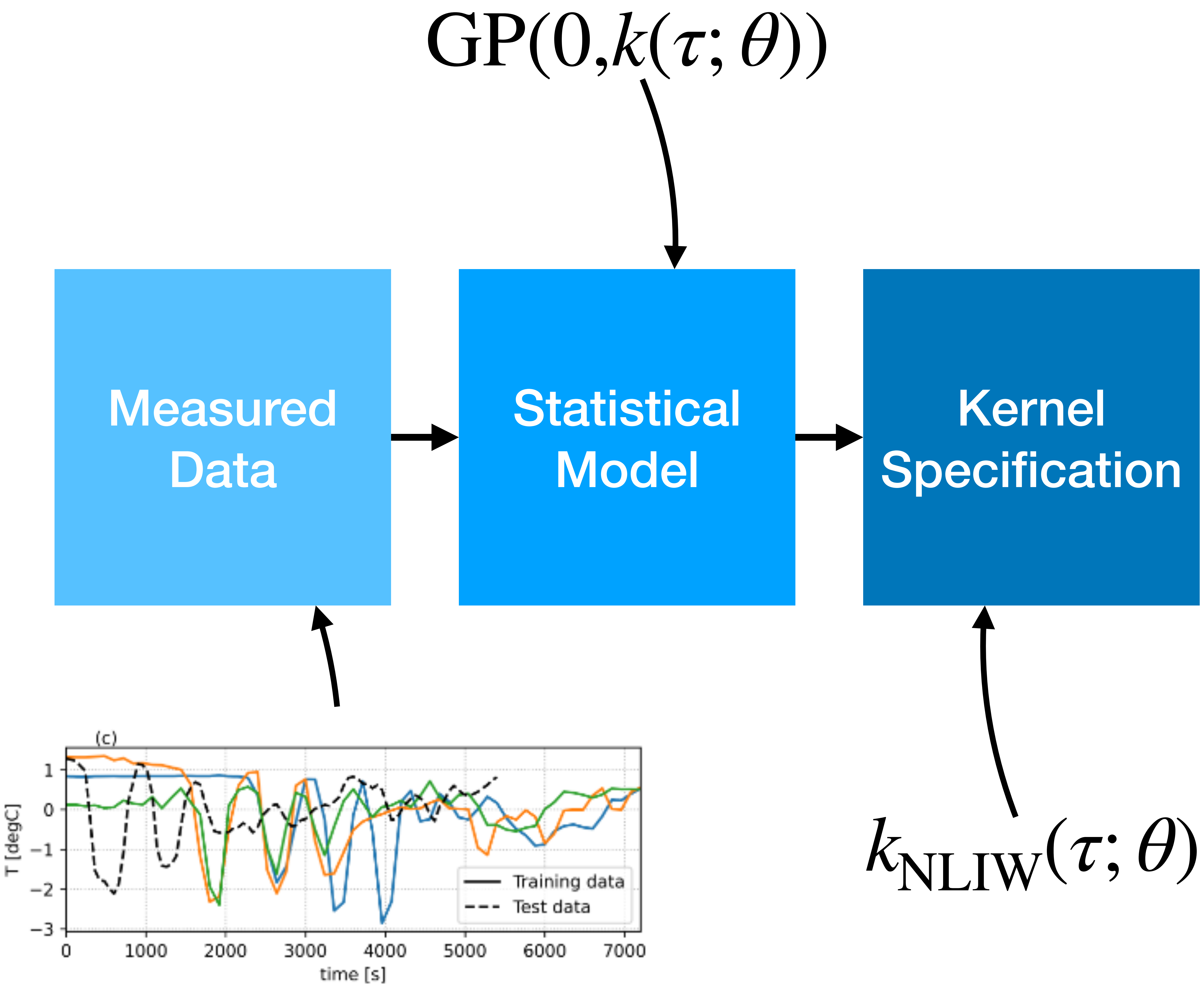


Measured Data

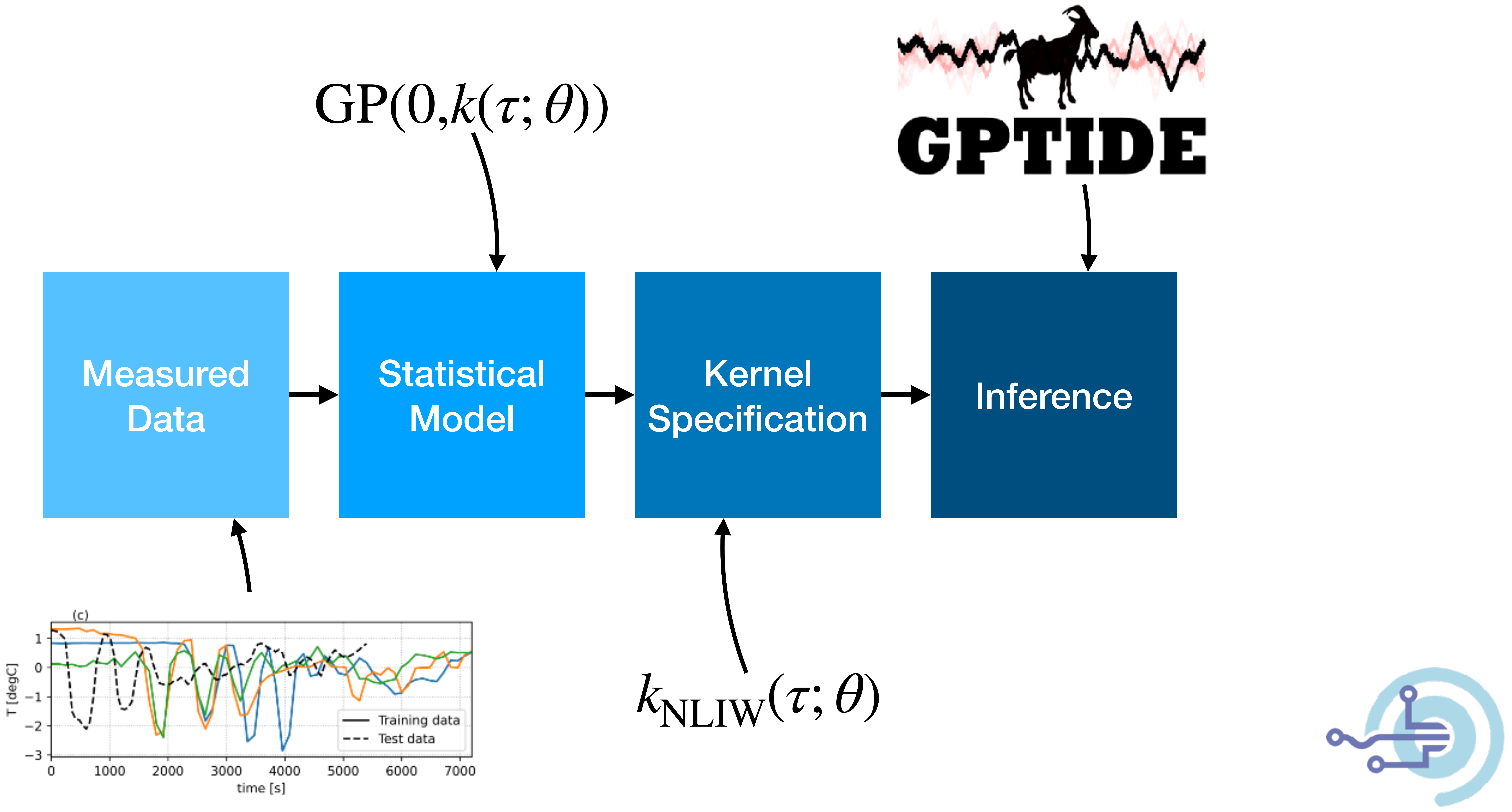




$\mathrm{GP}(0, k(\tau; \theta))$
Measured Data
Statistical Model
(c)
T [degC]
time [s]
Training data
Test data



$\mathrm{GP}(0, k(\tau; \theta))$
Measured Data
Statistical Model
Kernel Specification
$k_{\mathrm{NLIW}}(\tau; \theta)$
(c)
T [degC]
time [s]
Training data
Test data


GP(0,k(τ; θ))
GPTIDE
Measured Data
Statistical Model
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(c)
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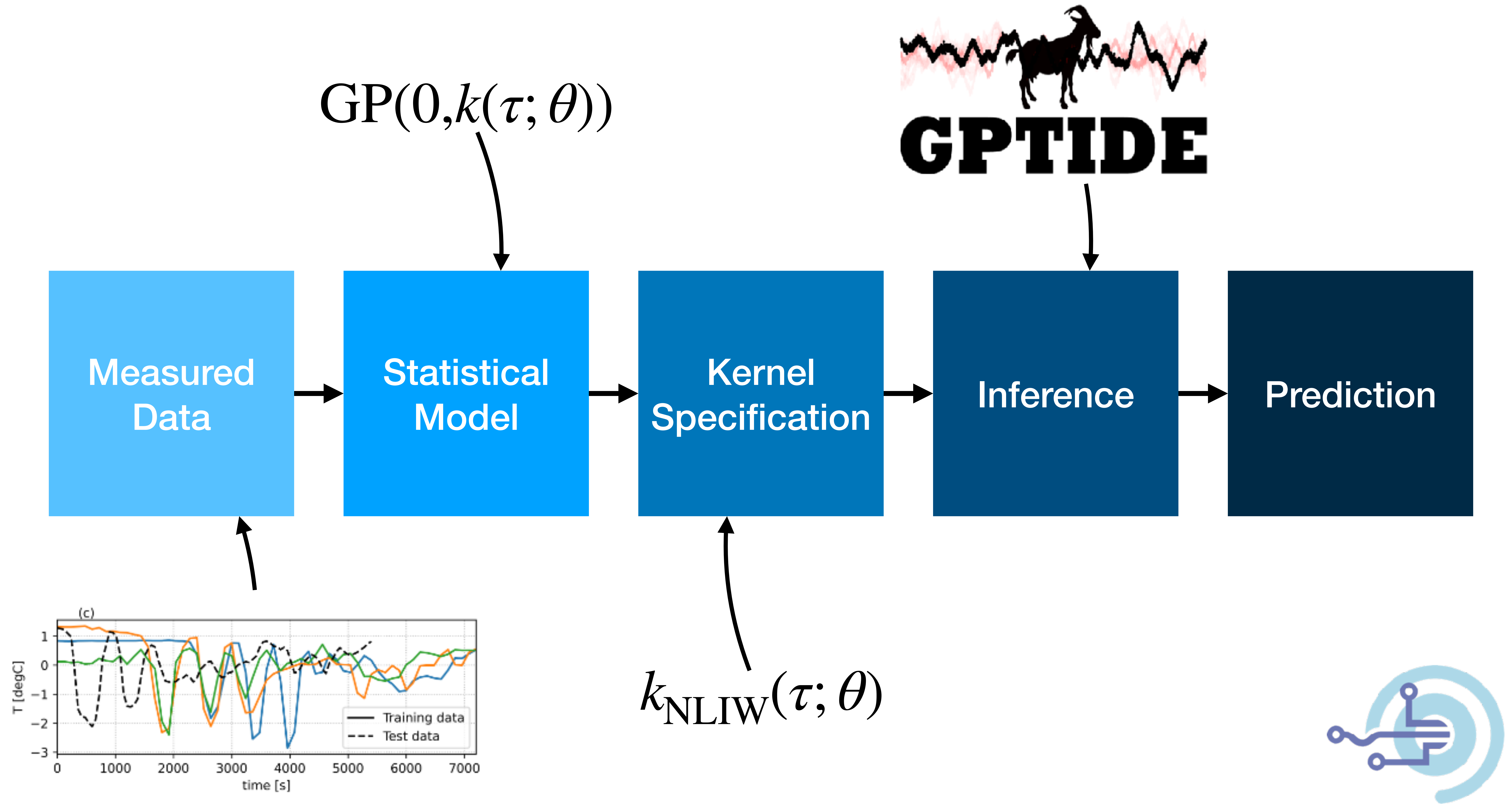

$GP(0, k(\tau; \theta))$
GPTIDE
Measured Data
Statistical Model
Kernel Specification
Inference
Prediction
$k_{NLIW}(\tau; \theta)$
(c)
T [degC]
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Test data

GP(0,k(τ; θ))
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Inference
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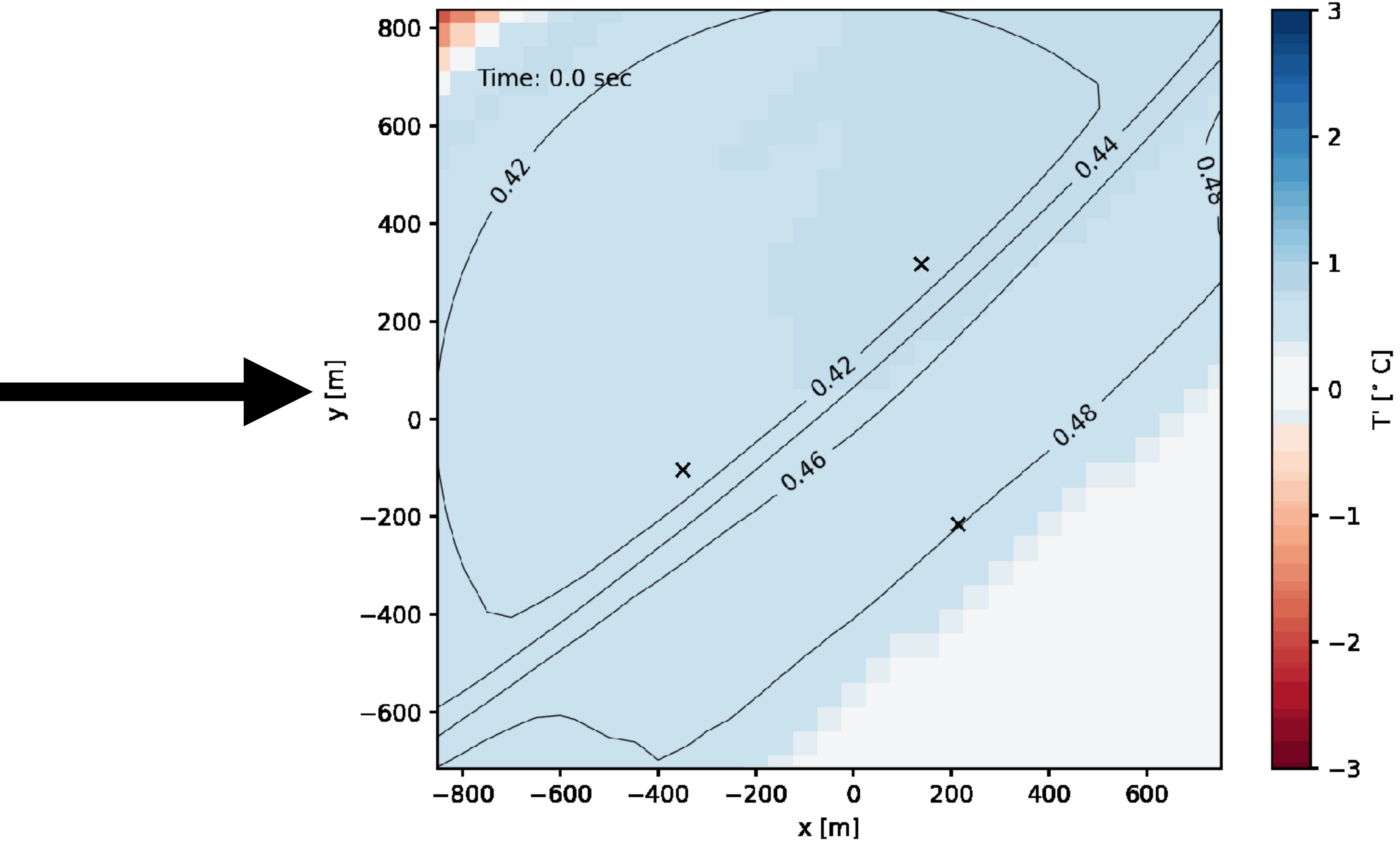
(c)
T [degC]
time [s]
Training data
Test data
k_NLIW(τ; θ)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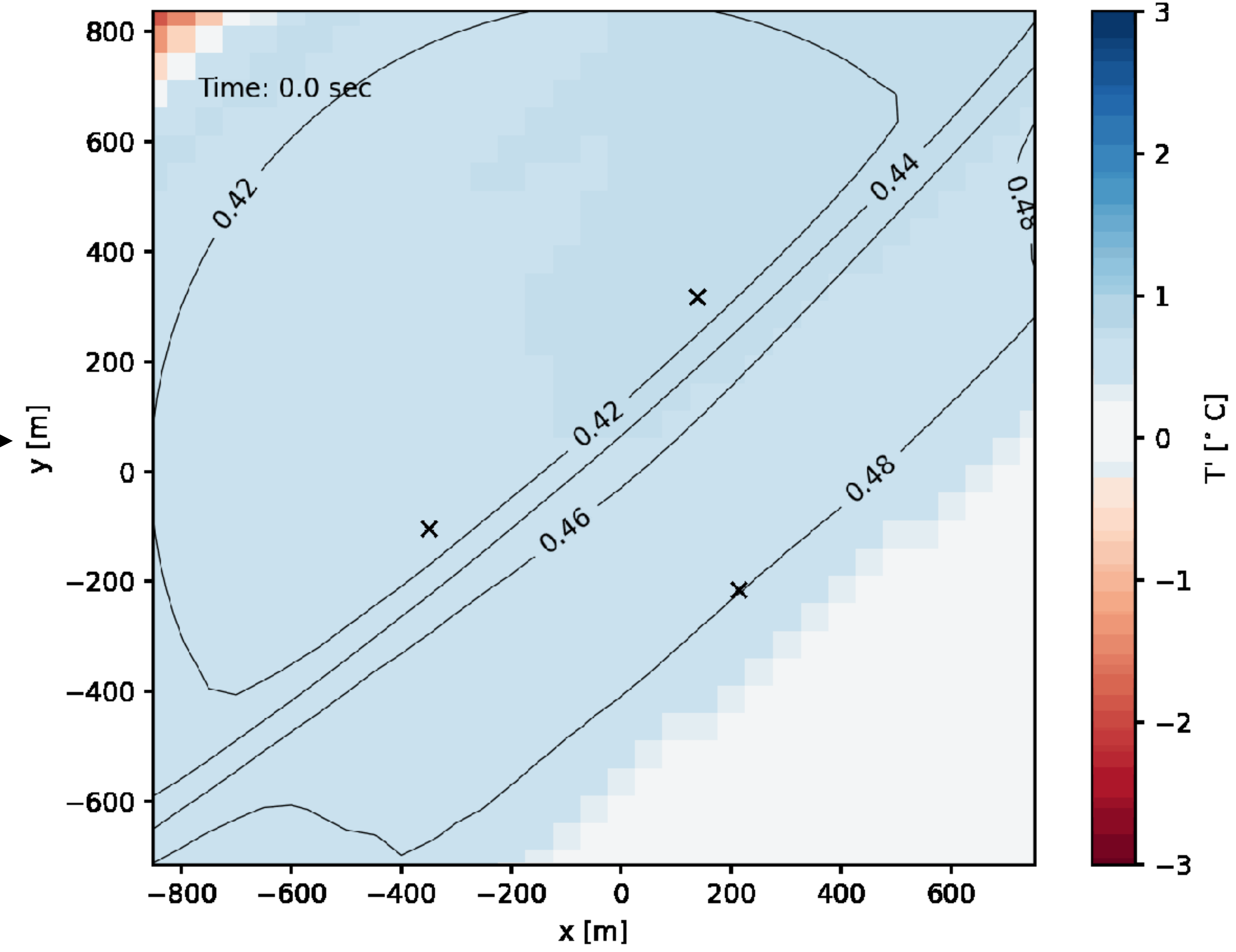
T' [° C]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y [m]
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T' [° C]
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−600
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−600
−400
−200
0
200
400
600
3
2
1
0
−1
−2
−3

Where to from here

- Characterise field profiles of NLIWs
- Automated detection of NLIW events from the background process
- Optimise code to run in real time for operations
- Include parameters to model NLIW curvature (only detectable by larger arrays)
- Test on other arrays and for longer periods of time (will require some computational tricks)
- Extend to moorings that observe currents through depth

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- *gptide* code available at **github.com/TIDE-ITRH/gptide**
- *gptide* tutorials available at **gptide.readthedocs.io**
- Email methodological queries to **lachlan.astfalck@uwa.edu.au**
- Email software queries to **andrew.zulberti@uwa.edu.au**
- Visit **tide.edu.au** for more information about the ARC Research Hub for Transforming energy Infrastructure through Digital Engineering (TIDE)