Applications of Gaussian Processes in Oceanography

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With contributions from Will Edge, Andrew Zulberti, Matt Rayson, Aurelien Ponte, Michael Bertolacci, Nicole Jones, Ed Cripps

Physical Oceanography

Hydrodynamics of Sea-surface Structures

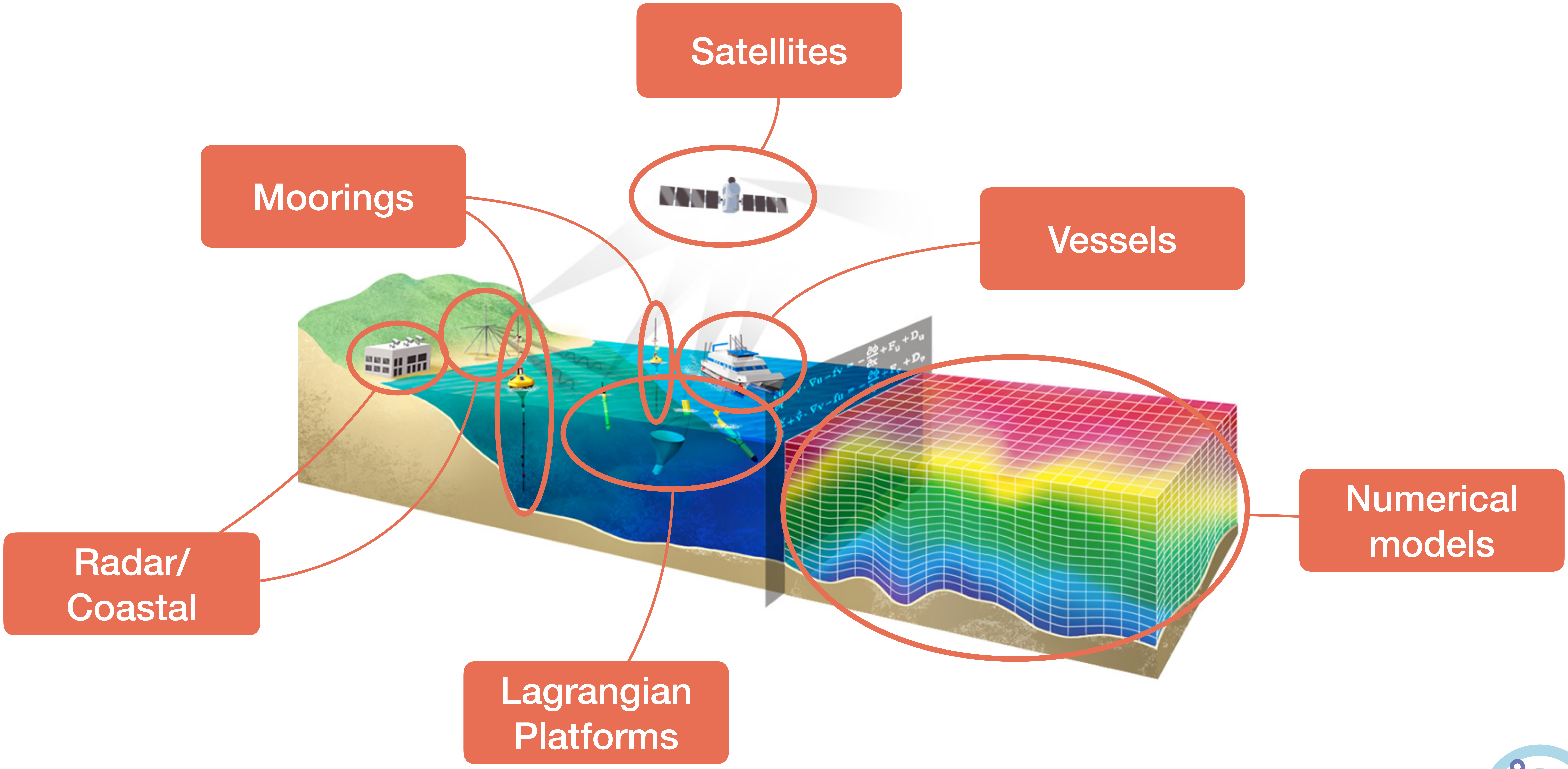
Sea-bed Geotechnics

Data Science

TIDE

ARC Research Hub for
Transforming energy Infrastructure
through Digital Engineering

Image credit: https://stephengriffies.github.io/models/



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INDONESIA
TIMOR SEA
ARAFURA SEA
PAPUA NEW GUINEA
CORAL SEA
CORAL SEA ISLANDS TERRITORY
INDIAN OCEAN
Darwin
NORTHERN TERRITORY
Cairns
Mount Isa
QUEENSLAND
Mackay
AUSTRALIA
Alice Springs
Rockhampton
Gladstone
Tropic of Capricorn
WESTERN AUSTRALIA
Bundaberg
Maryborough
Maroochydore
Brisbane
SOUTH AUSTRALIA
Toowoomba
Gold Coast
Lismore
NEW SOUTH WALES
Geraldton
Kalgoorlie-Boulder
Broken Hill
Tamworth
Port Macquarie
Perth
Whyalla
Dubbo
Maitland
Newcastle
Rockingham
Bathurst
Mildura
Orange
Gosford
PACIFIC OCEAN
Bunbury
Adelaide
Goulburn
Sydney
Wagga Wagga
Wollongong
Bendigo
Canberra
Queanbeyan
Shepparton
Ballarat
AUSTRALIAN CAPITAL TERRITORY
Mount Gambier
Melbourne
Warrnambool
Geelong
VICTORIA
INDIAN OCEAN
TASMAN SEA
Devonport
TASMANIA
Hobart
130° 135° 140° 145° 150° 155°
10° 15° 20° 25° 30° 35° 40°
0 150 300 mi
0 200 400 km
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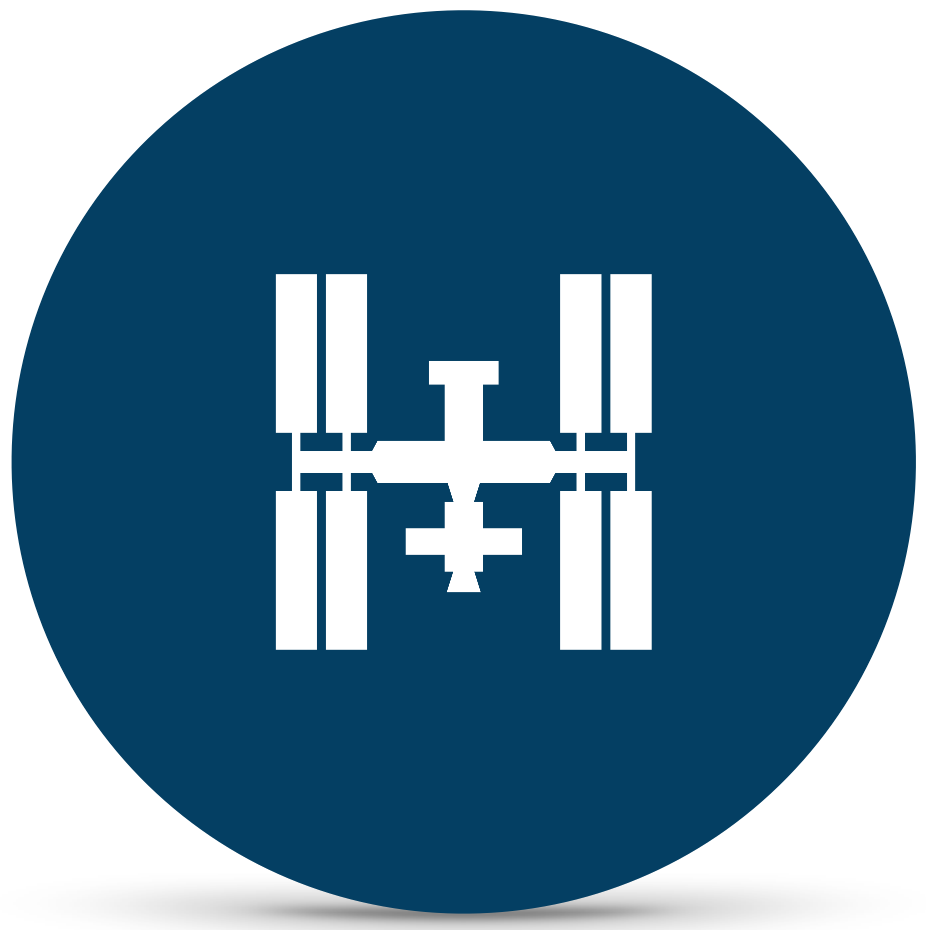
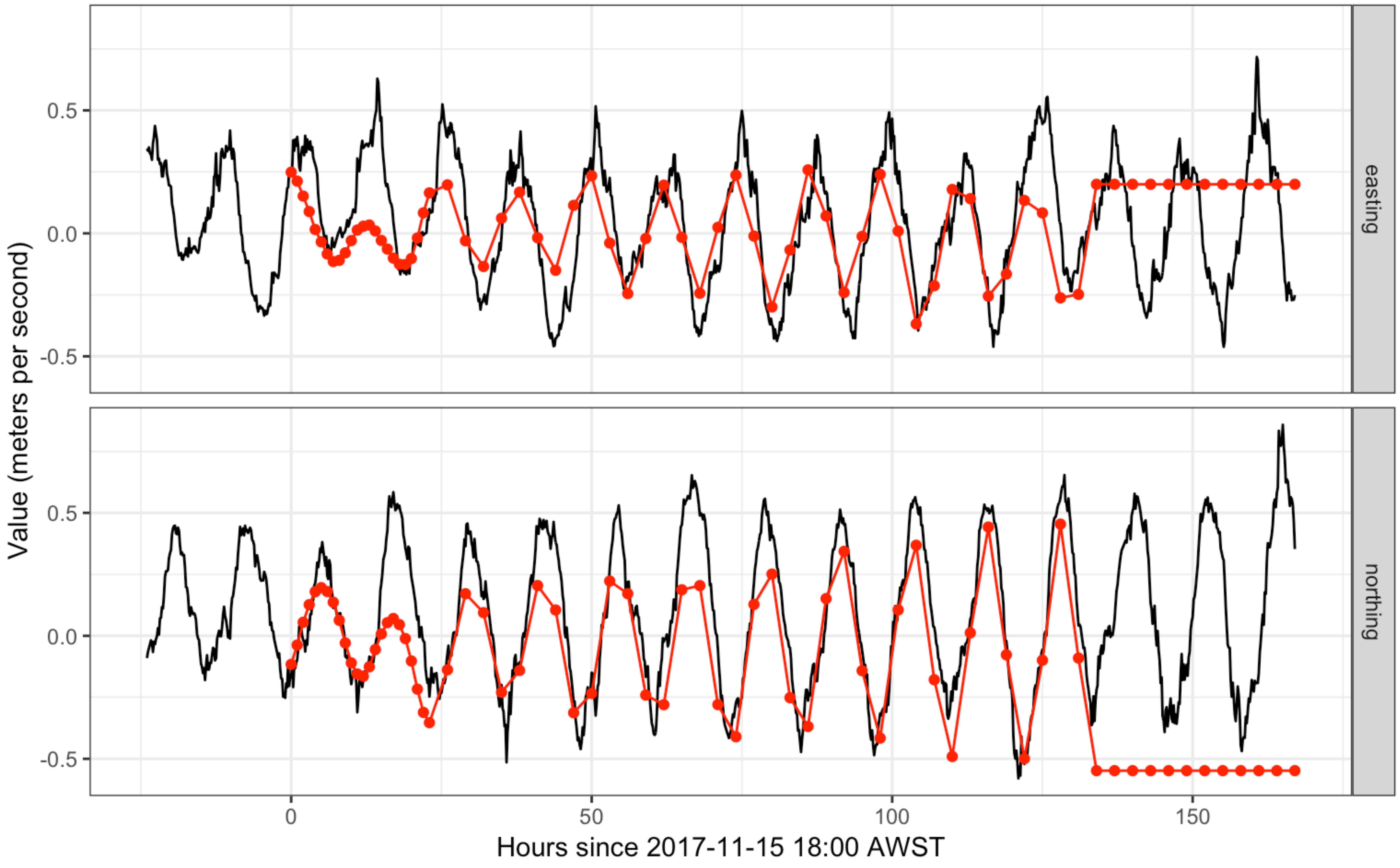
Warm-up GPs



Physics Informed Covariance

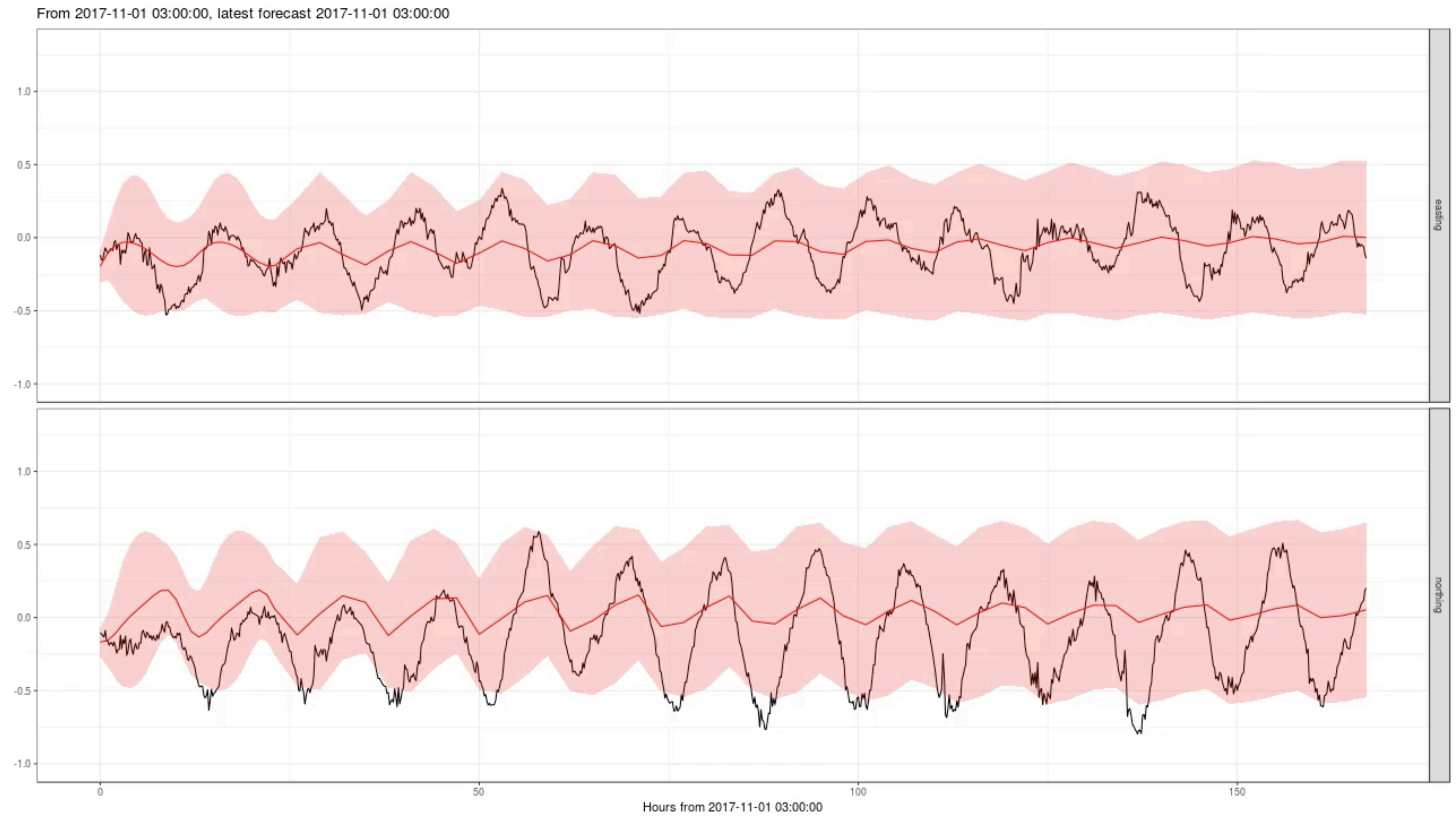
Merging Data

Surface Currents



MG Bertolacci, LC Astfalck, EJ Cripps. (2024). ‘Bayesian integration of surface current astronomic potential and stochasticity’. In preparation for Ocean Engineering.

Surface Currents



$$k(\,\cdot\,,\cdot\,) = \text{periodic}_{\text{M2}} + \text{periodic}_{\text{S2}} + \text{periodic}_{\text{M1}} + \text{periodic}_{\text{S1}} + \text{matern}_{\text{short}} + \text{matern}_{\text{long}}$$

$$(u, v)_t \sim \text{GP}\left(X\beta, K \otimes k(\,\cdot\,,\cdot\,)\right)$$

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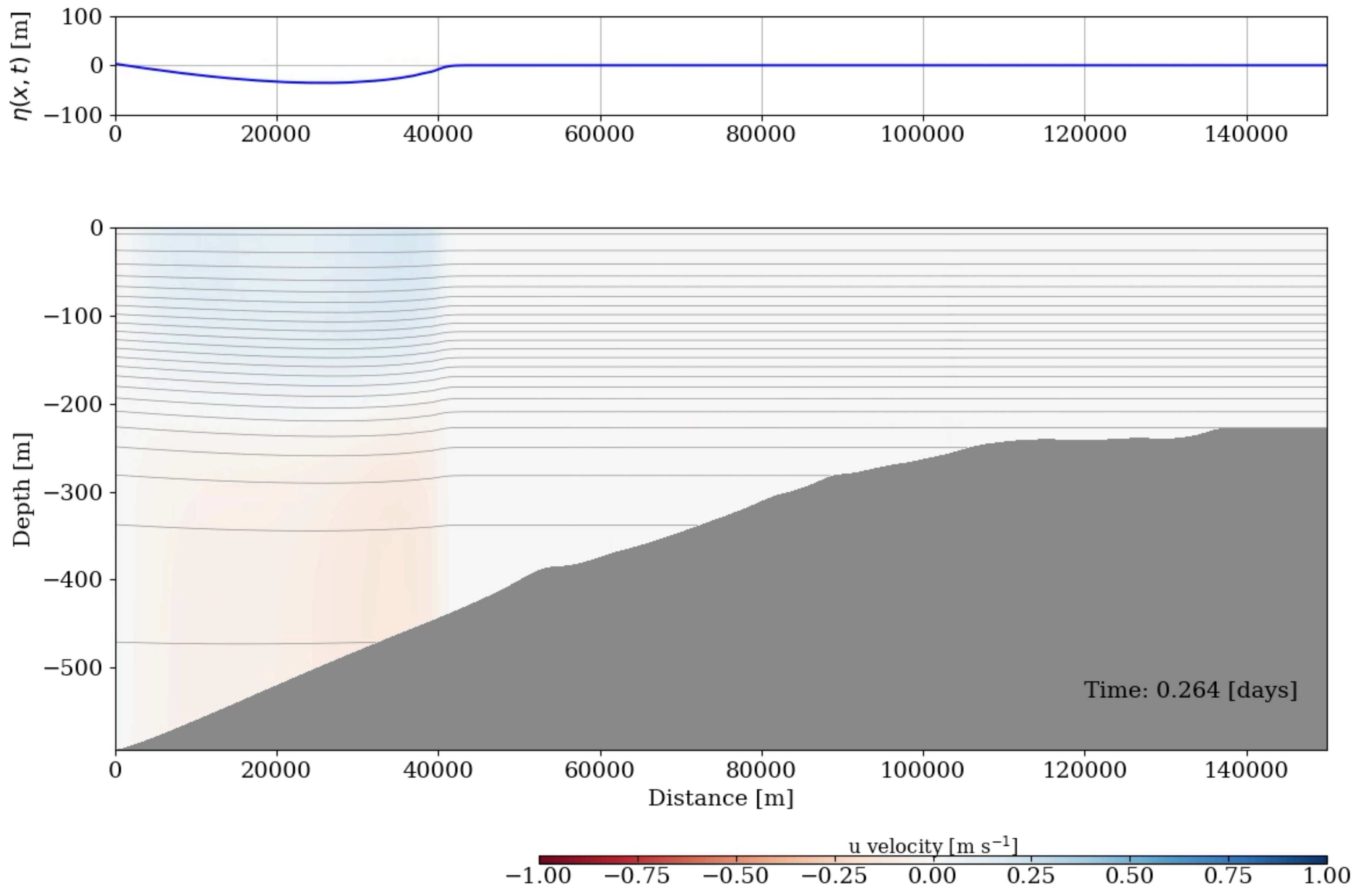
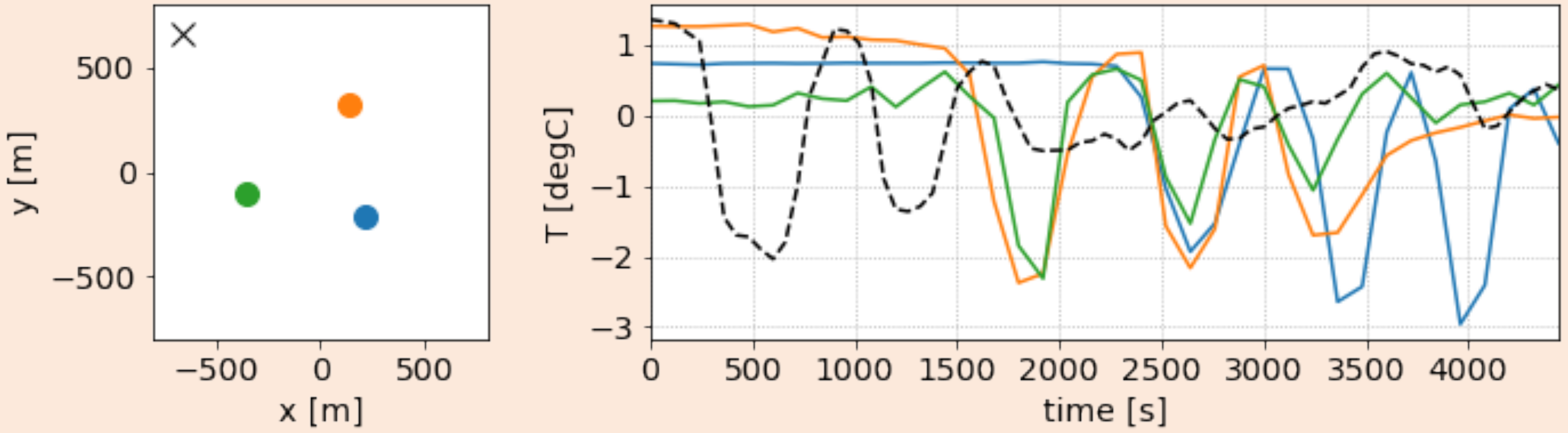
Non-linear Internal Waves

Numerically Modelled:



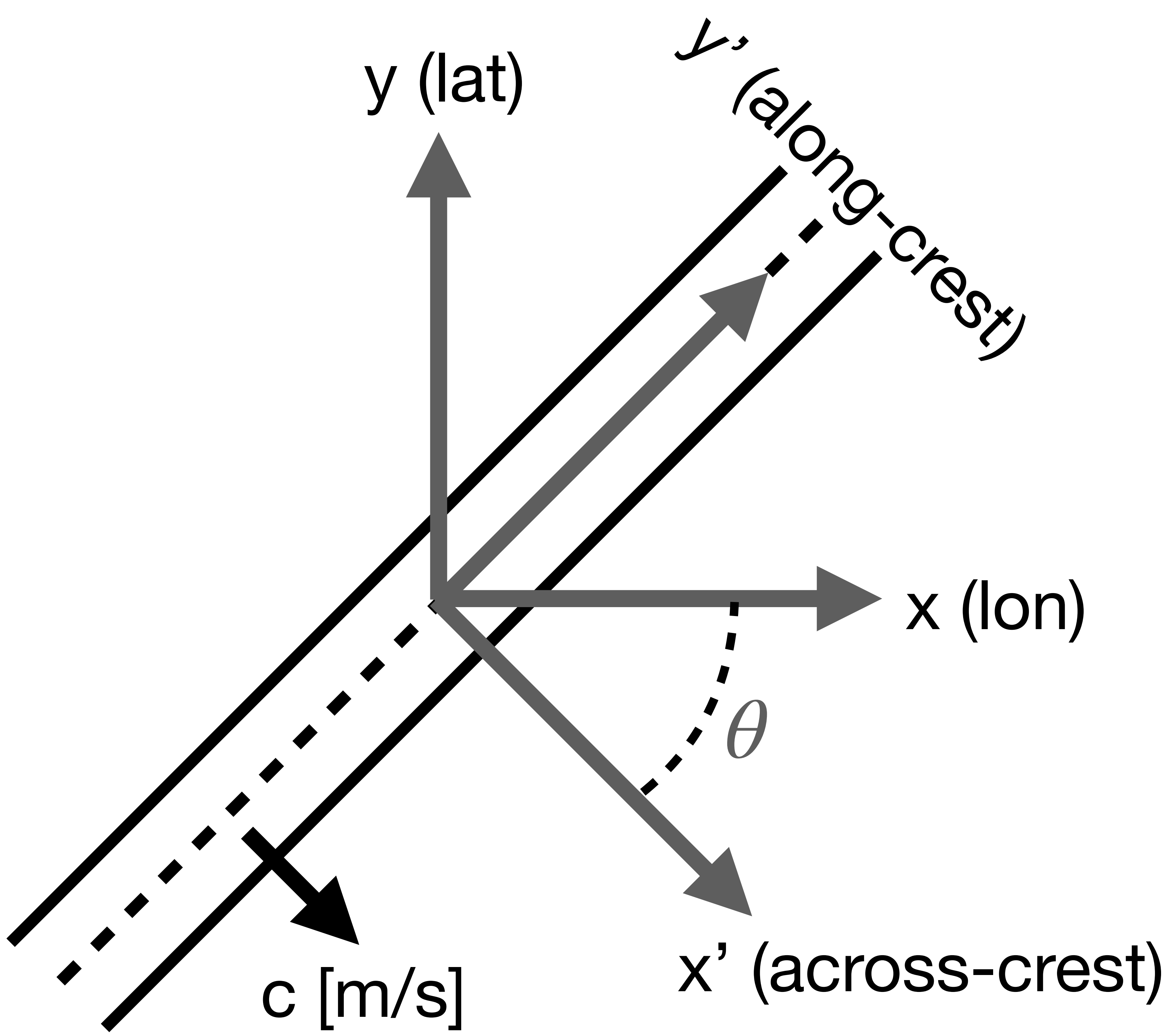
Kimberley Internal Soliton, Sediment and Mixing Experiment (KISSME)

Observed:

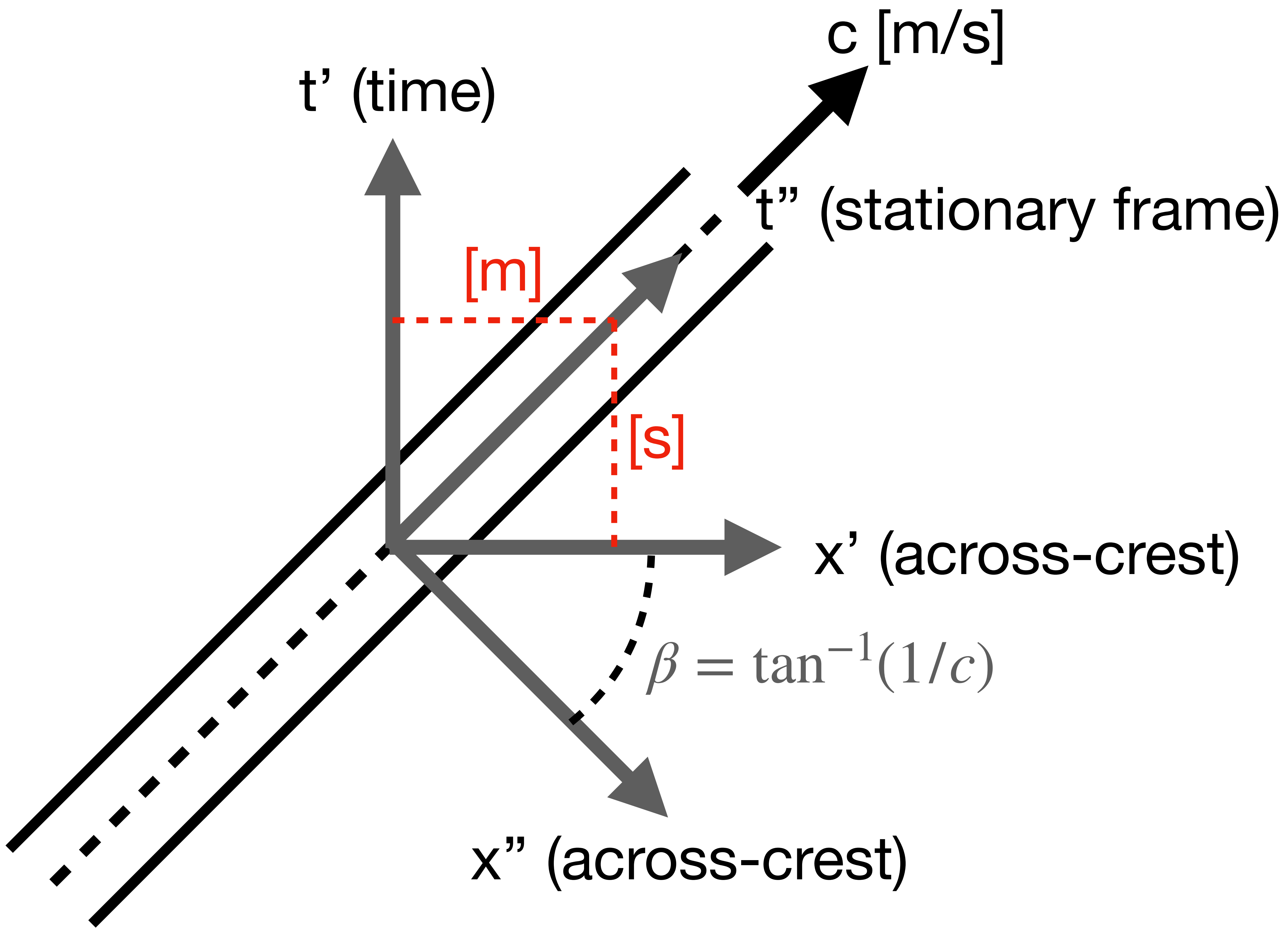


AP Zulberti, NL Jones, GN Ivey. (2020). Observations of enhanced sediment transport by nonlinear internal waves. *Geophysical Research Letters*

Non-linear Internal Waves



Latitude/Longitude Projection

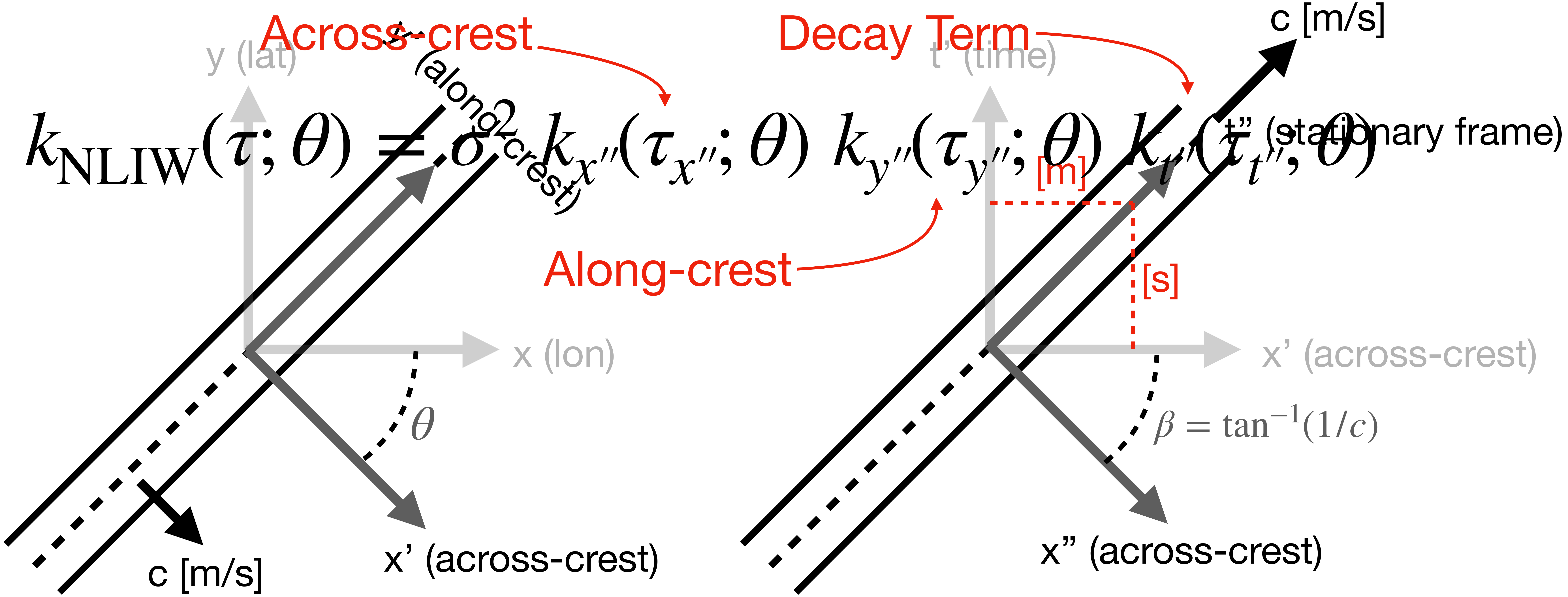


Across-crest/Time Projection

MD Rayson, LC Astfalck, AP Zulberti, EJ Cripps, NL Jones. (2024). 'Inferring nonlinear internal wave properties from sparse observations using Gaussian process regression'. In preparation for JAMES.

Non-linear Internal Waves

$$k_{\mathrm{NLIW}}(\tau;\theta) = \sigma^2 k_{x''}(\tau_{x''};\theta)\; k_{y''}(\tau_{y''};\theta)\; k_{t''}(\tau_{t''};\theta)$$



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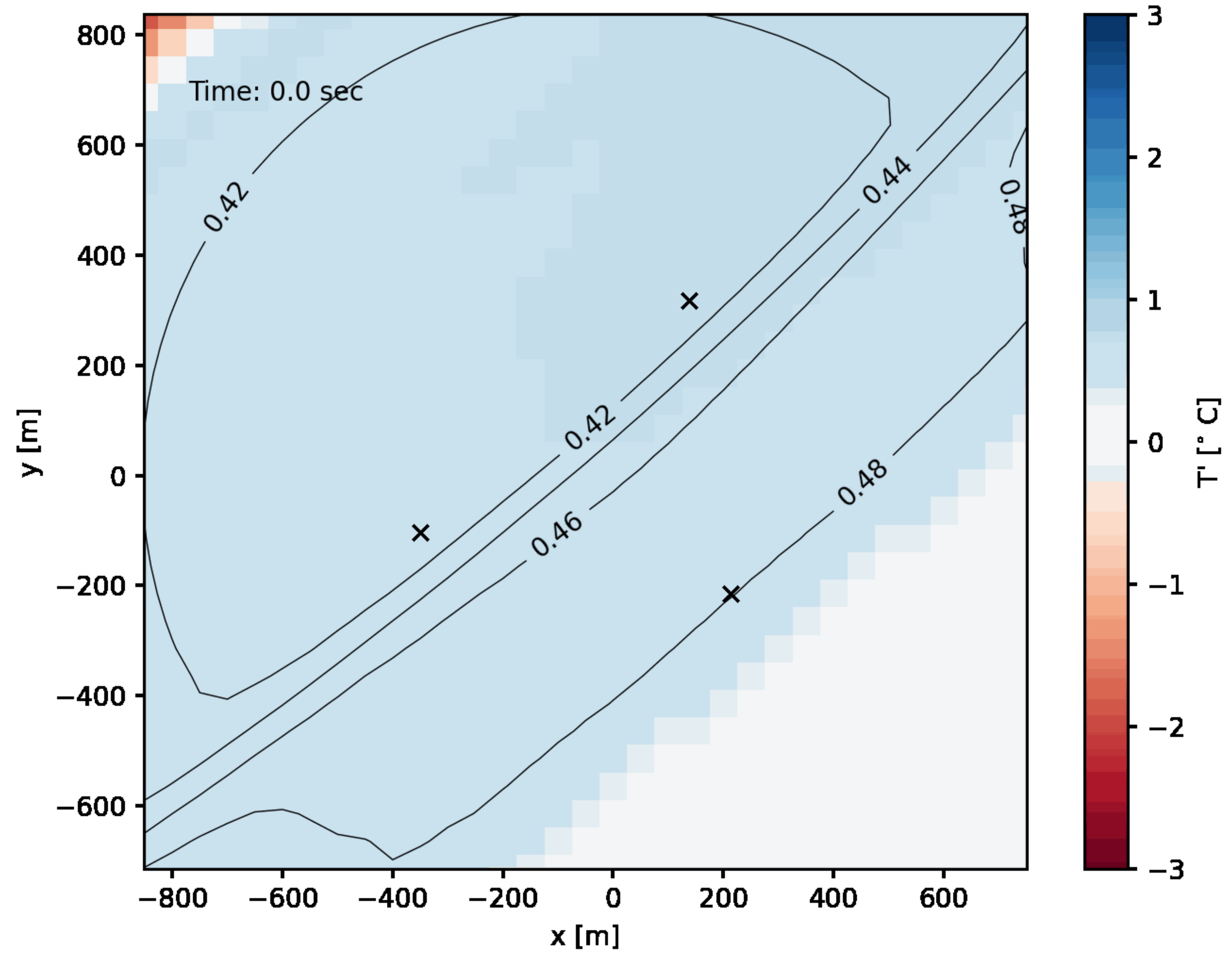
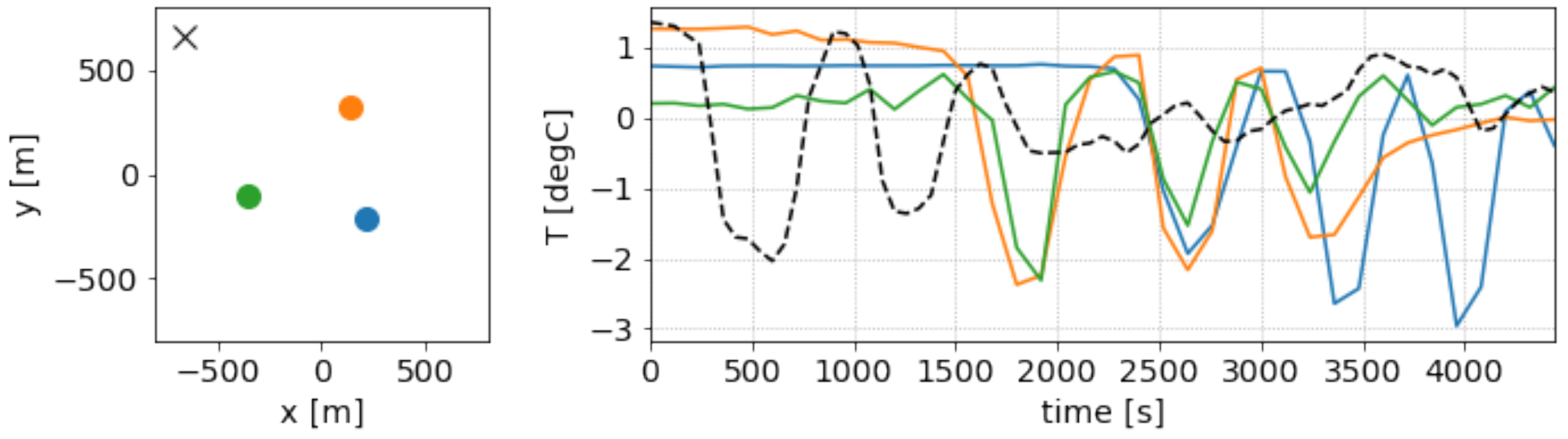
Non-linear Internal Waves





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The Helmholz kernel

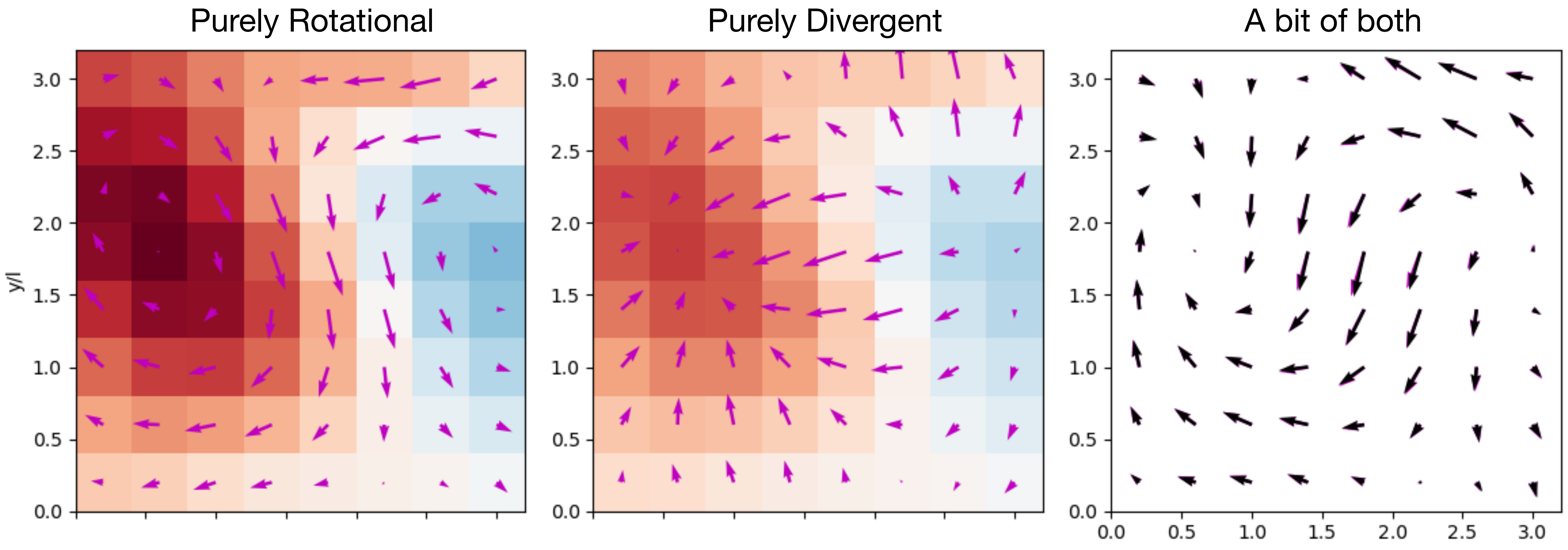
Streamfunction = Rotation

$$u = -\partial_y \psi + \partial_x \phi$$
$$v = \partial_x \psi + \partial_y \phi$$

Velocity potential = Divergence

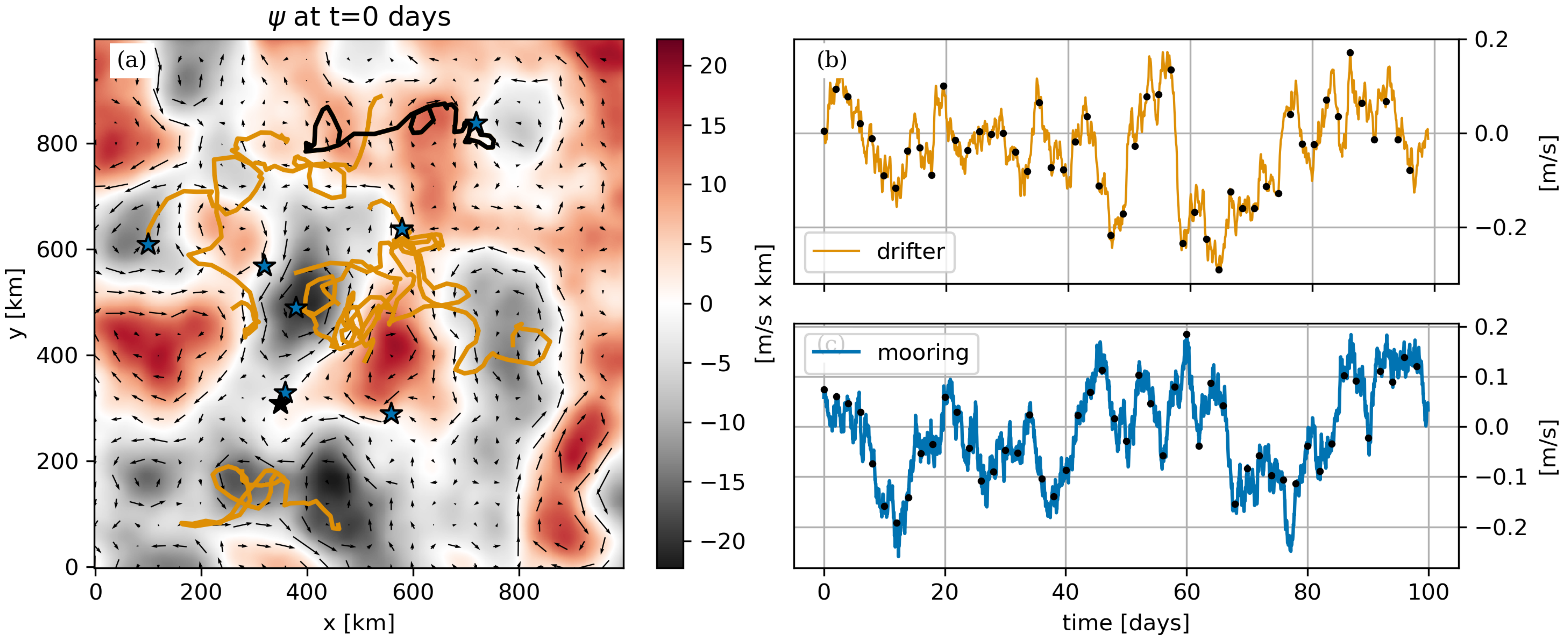
$$\longrightarrow$$

$$k_{uu} = -\partial_{yy} k_{\psi\psi} - \partial_{xx} k_{\phi\phi} + \partial_{xy} k_{\phi\psi} + \partial_{xy} k_{\psi\phi},$$
$$k_{vv} = -\partial_{xx} k_{\psi\psi} - \partial_{yy} k_{\phi\phi} - \partial_{xy} k_{\phi\psi} - \partial_{xy} k_{\psi\phi},$$
$$k_{uv} = \partial_{xy} k_{\psi\psi} - \partial_{xy} k_{\phi\phi} + \partial_{yy} k_{\phi\psi} - \partial_{xx} k_{\psi\phi},$$



Berlinghieri, Renato, et al. "Gaussian processes at the Helm (holtz): A more fluid model for ocean currents." *arXiv preprint arXiv:2302.10364* (2023).

Lagrangian Observations



- What features of the flow are better inferred with each observation platform?
- How do we parameterise our kernels for real-world data?

ALS Ponte, LC Astfalck, MD Rayson, AP Zulberti, NL Jones. (2024). 'Inferring flow energy, space and time scales: freely-drifting vs fixed point observations'. Nonlinear Processes in Geophysics.

Warm-up GPs

Physics Informed Covariance

Merging Data

The power spectral density

The power spectral density (PSD) describes the distribution of power/variance into sinusoidal frequencies that describe the signal

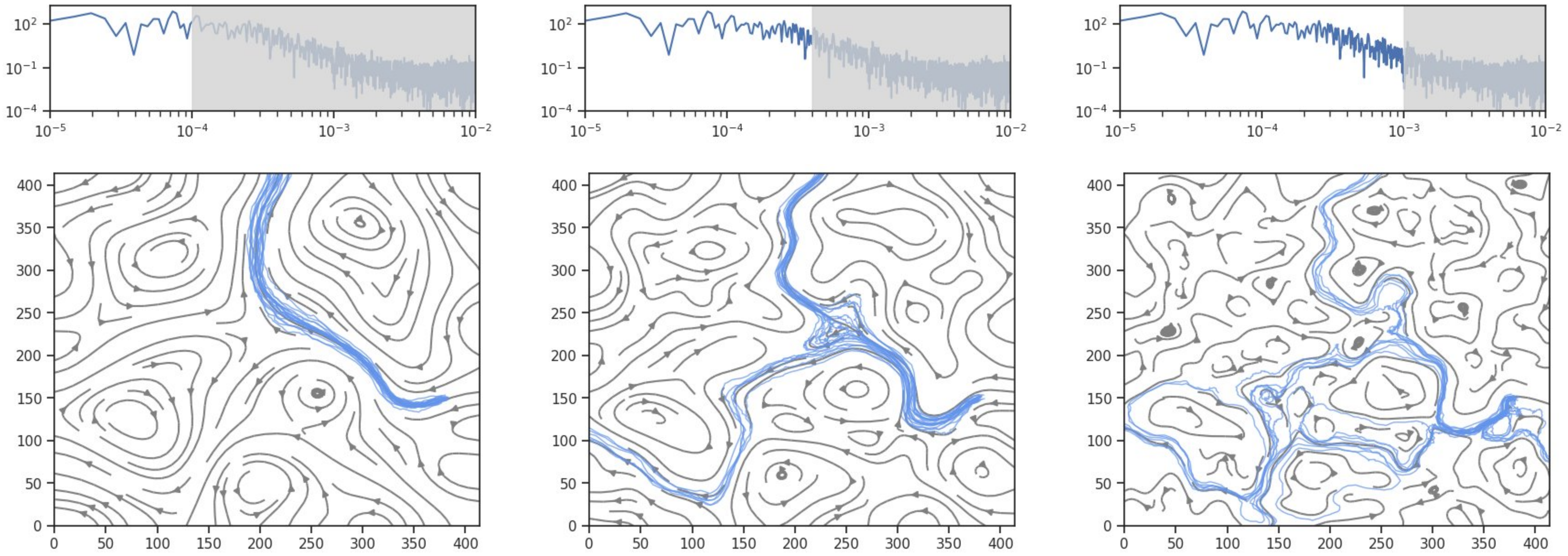
For *most* $k(\tau)$ there exists a power spectral density $f(\omega)$ so that

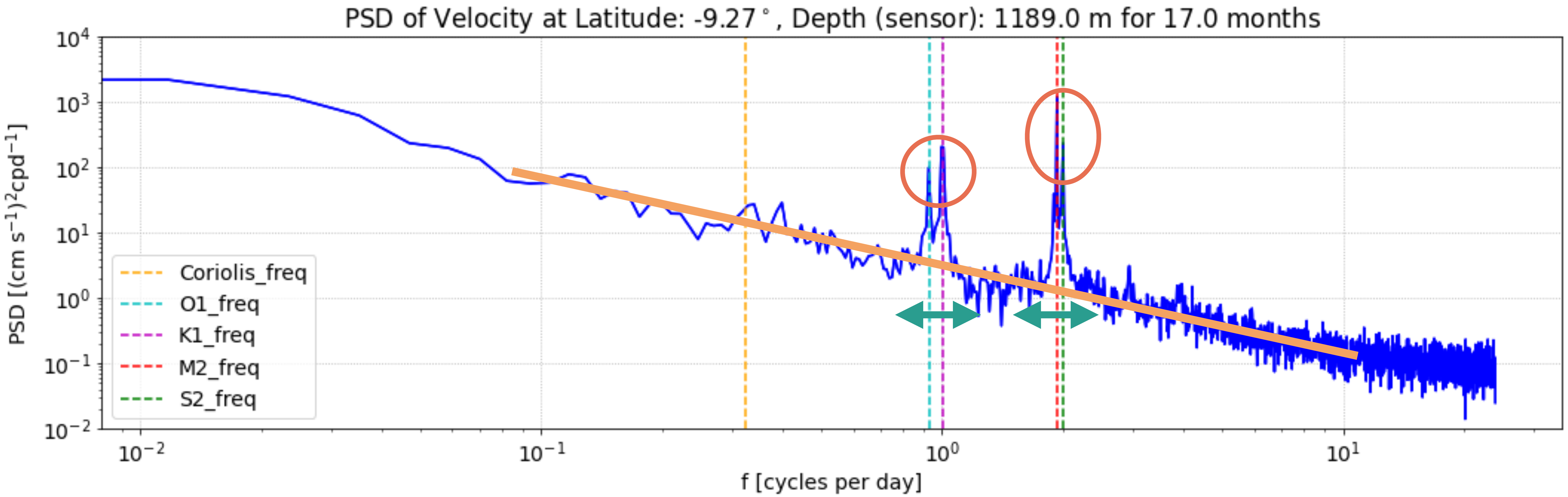
$$f(\omega) = \sum_{\tau=\infty}^{\infty} k(\tau) e^{-i\omega\tau}, \qquad k(\tau) = \int_{-1/2}^{1/2} f(\omega) e^{i\omega\tau} \, \mathrm{d}\omega$$

are Fourier pairs.

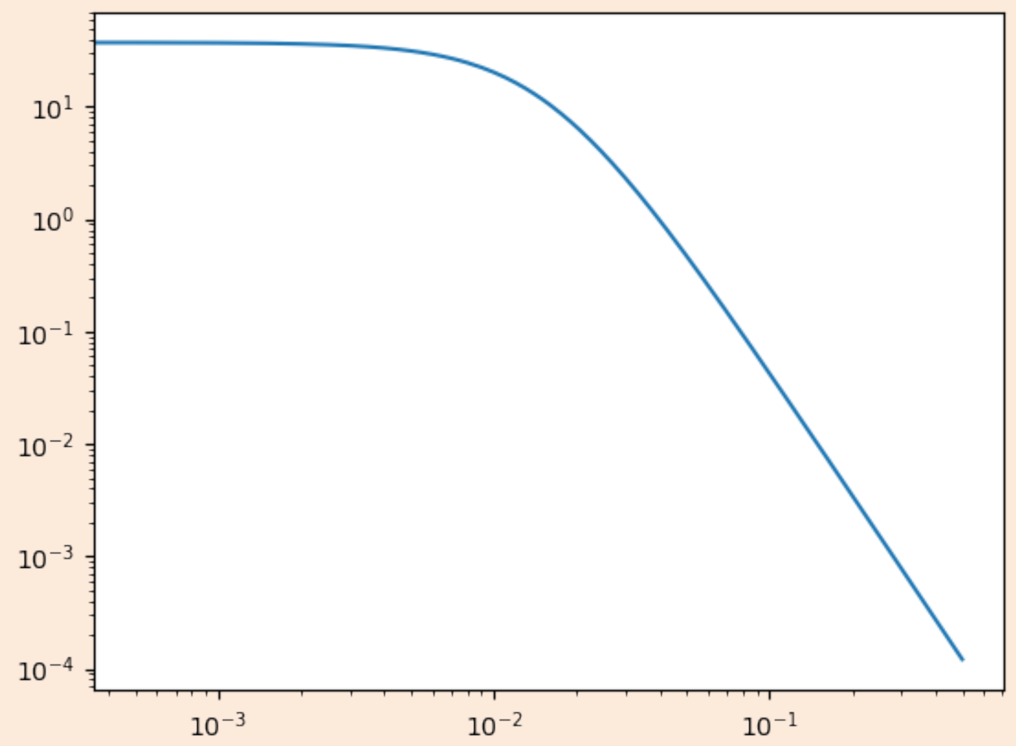
We may describe a process either by its ACF or PSD

What if we get it wrong?



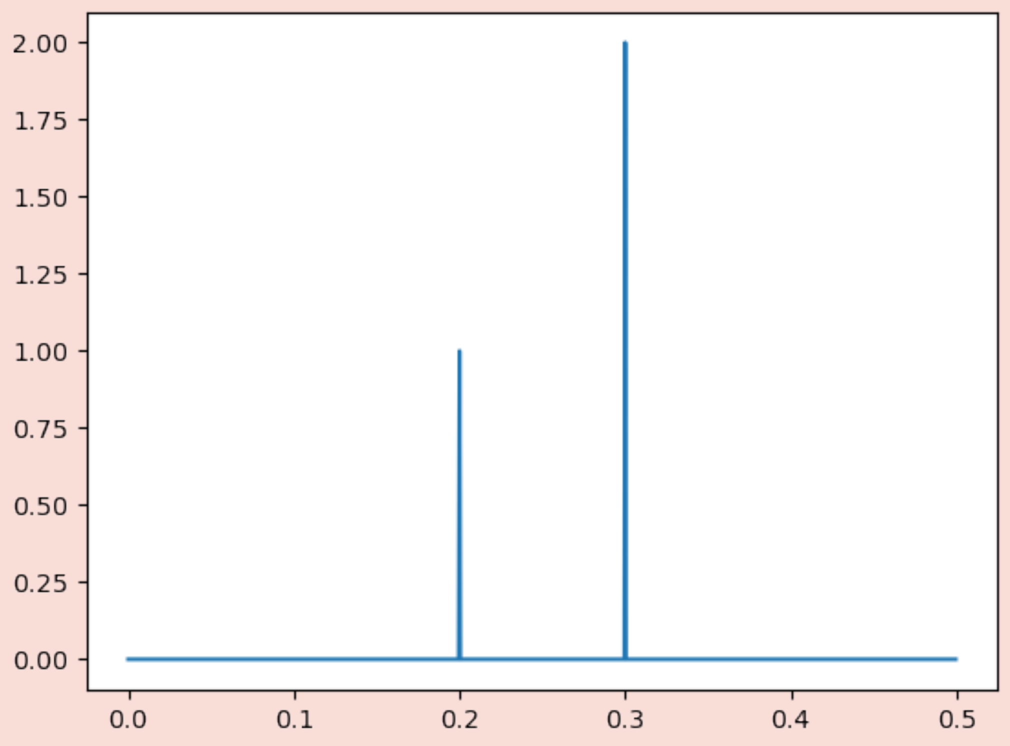


Background energy continuum



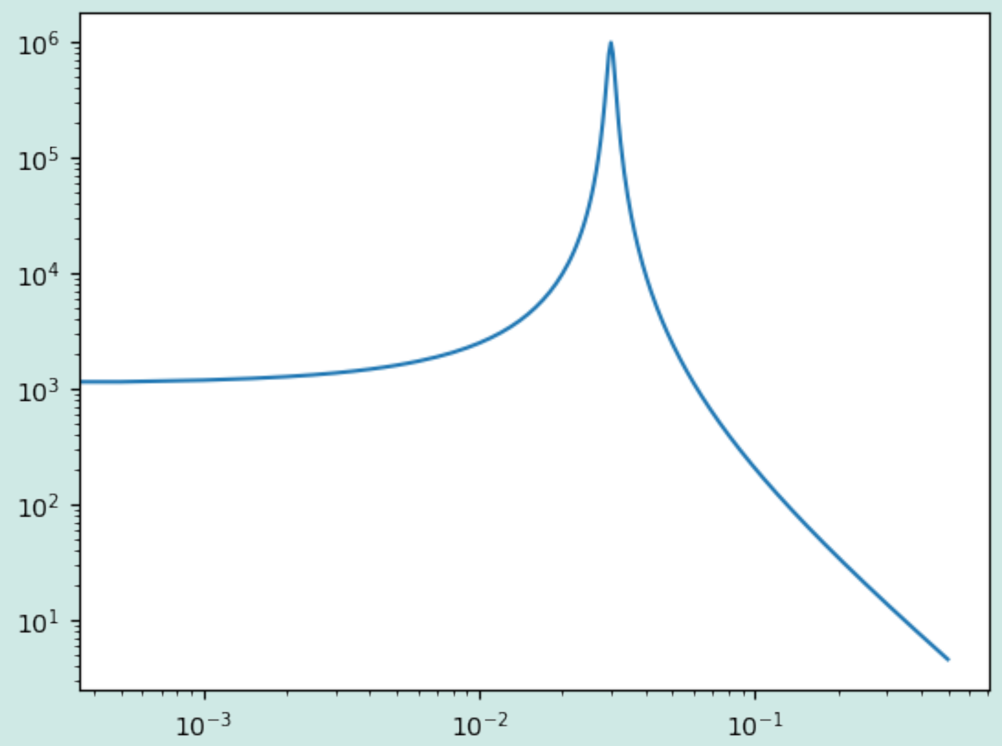
Matérn?

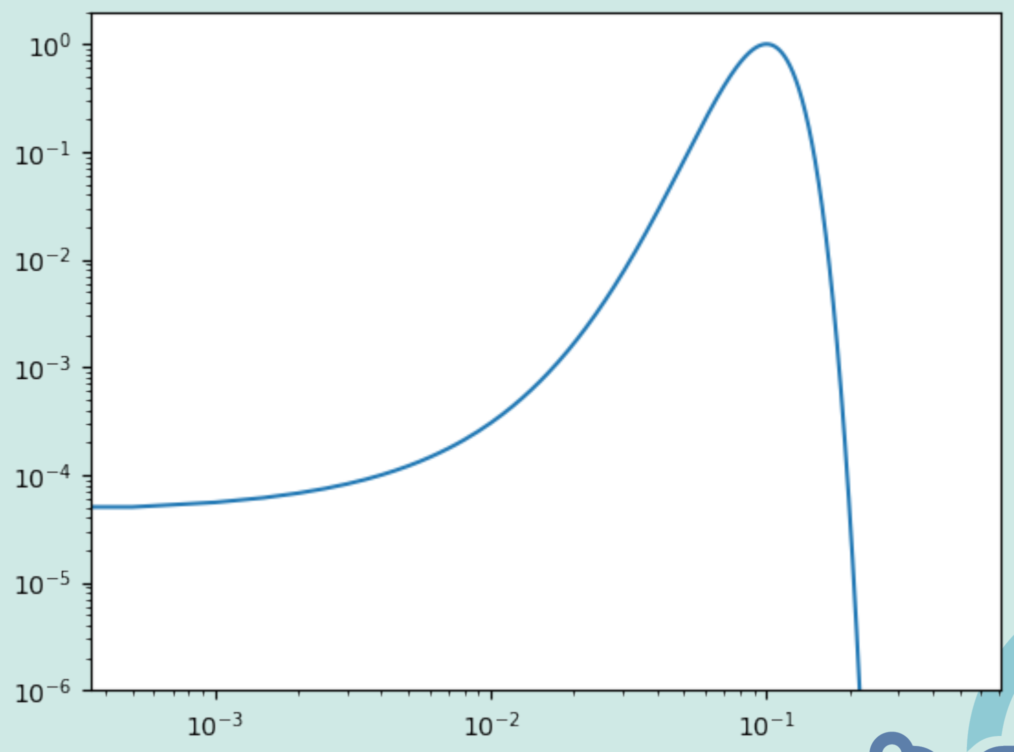
Phase locked tides



Harmonic Analysis

Non-phase locked tides

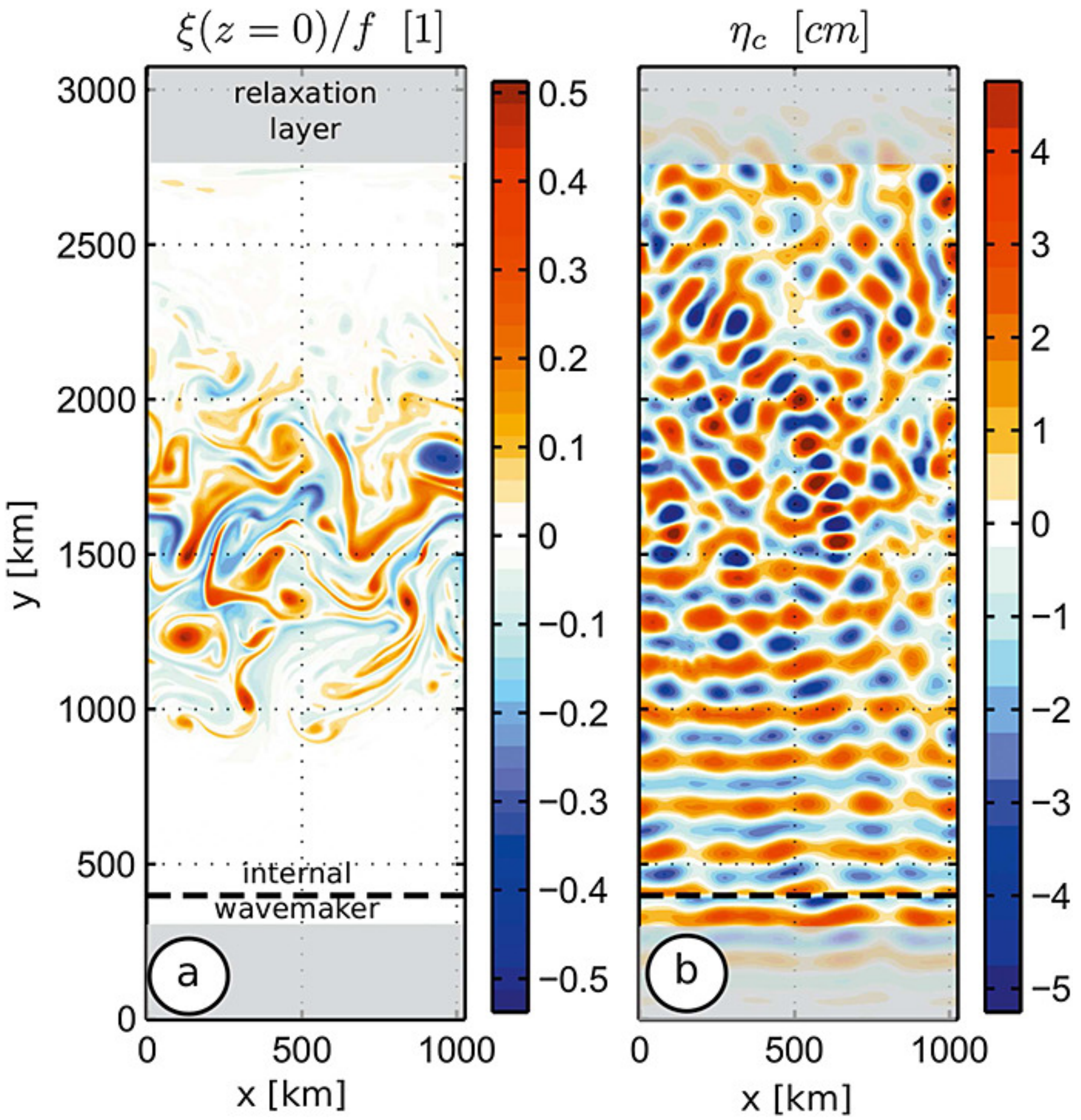




Lorenzian?

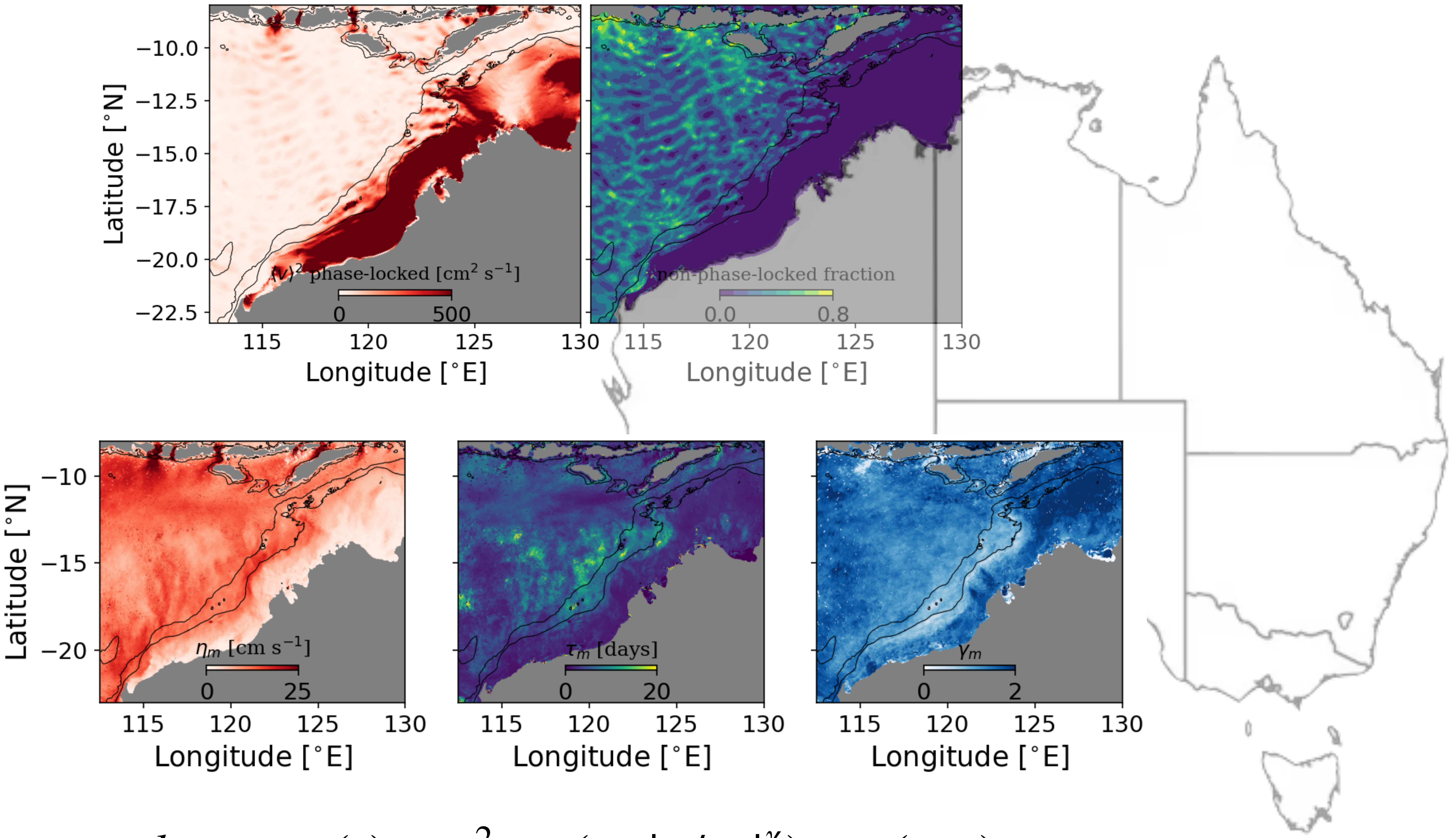
Gaussian?

Wave Direction



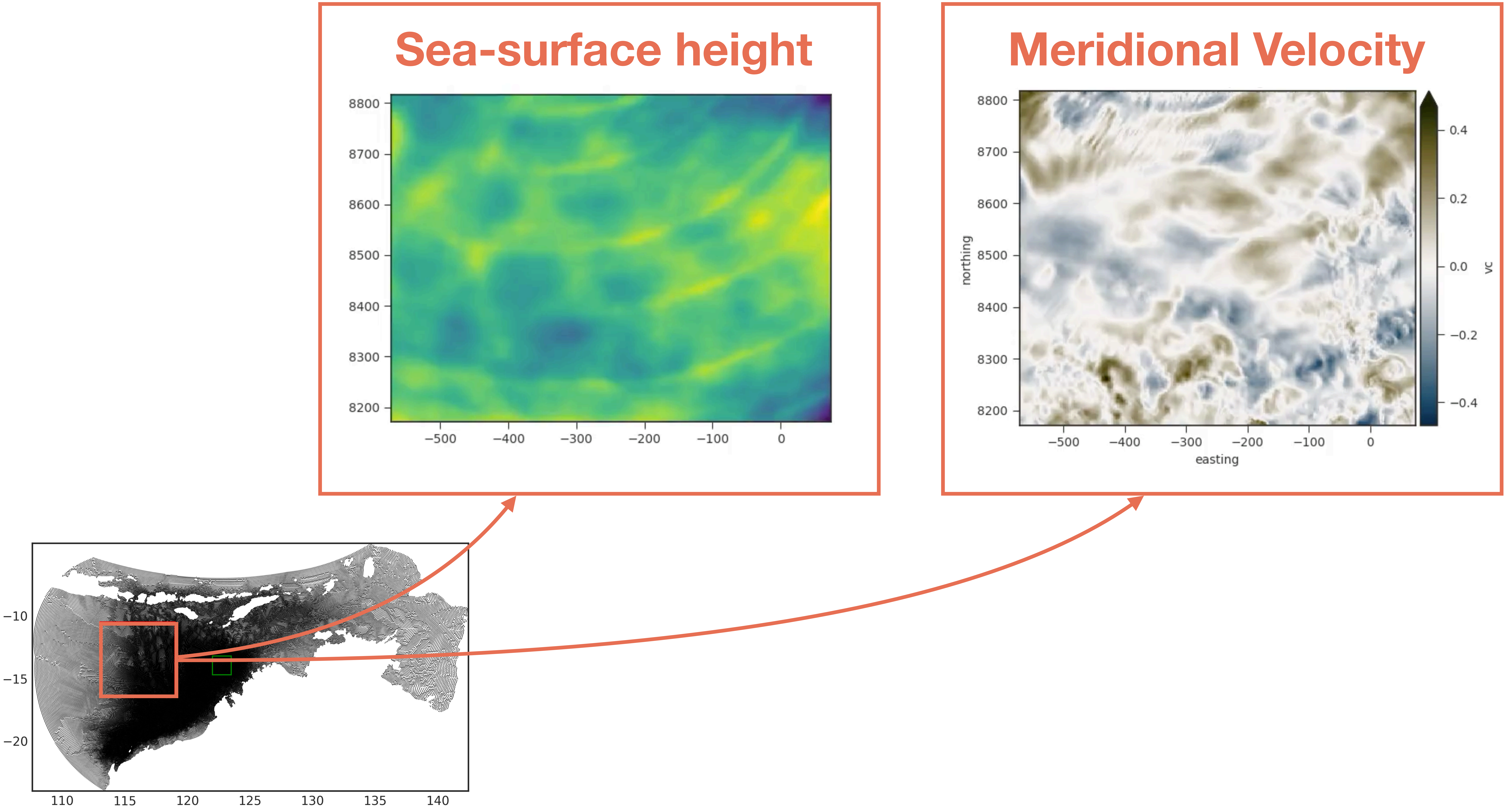
Increasing Incoherence

Ponte, A. L., & Klein, P. (2015). 'Incoherent signature of internal tides on sea level in idealized numerical simulations'. *Geophysical Research Letters*.



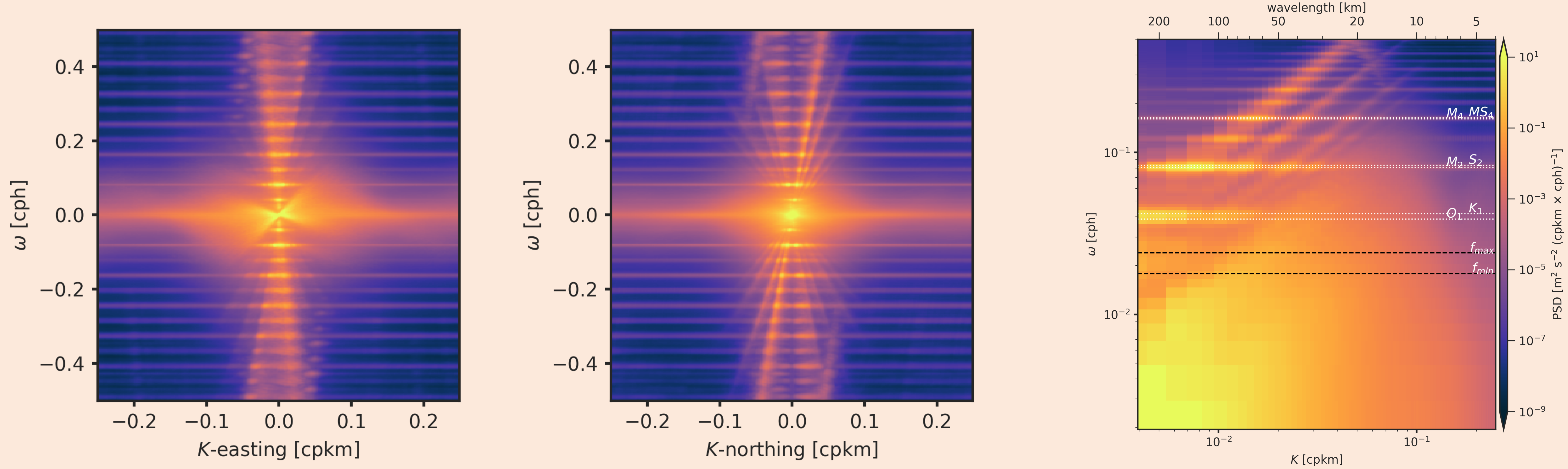
$$k_{\text{broadening}}(\tau) = \eta^2 \exp(-|\tau/\tau_{\text{d}}|^{\gamma}) \ \cos(\omega_0 \tau)$$

MD Rayson, LC Astfalck, ALS Ponte, AP Zulberti, NL Jones. (2024). 'Spectral model parameter estimation for non-phase-locked internal tides in a mesoscale eddy field'. Submitted to *JGR Oceans*.

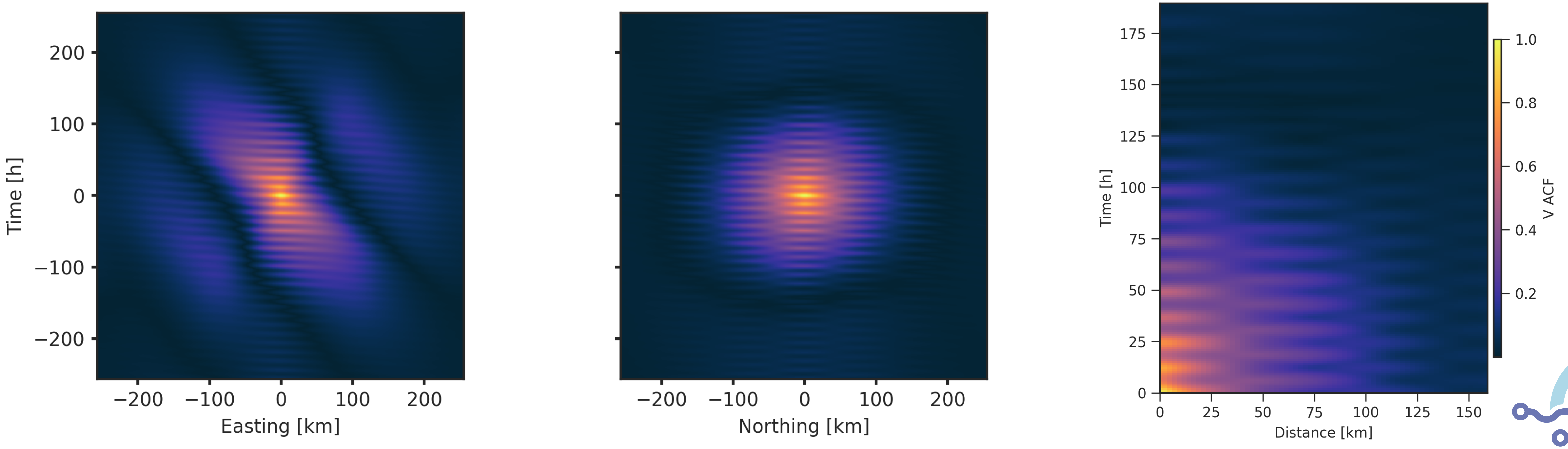


MD Rayson, NL Jones, GN Ivey, & Y Gong. (2021). A seasonal harmonic model for internal tide amplitude prediction. *JGR Oceans*.

PSDs

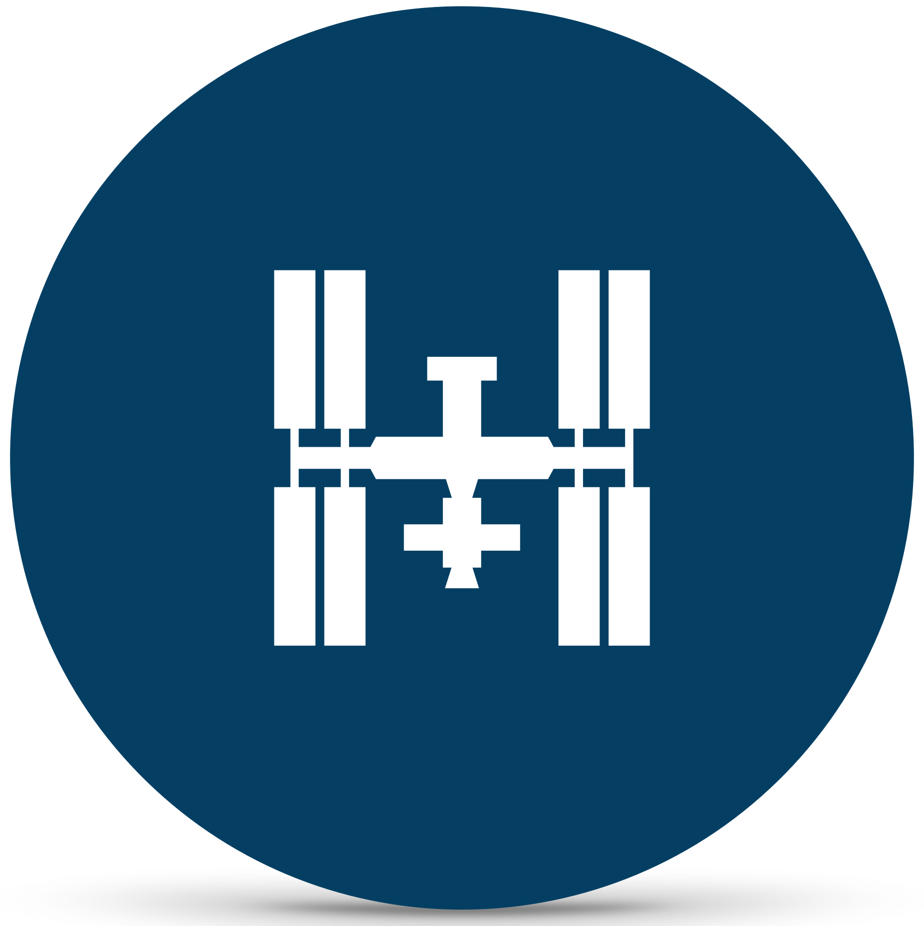


ACFs



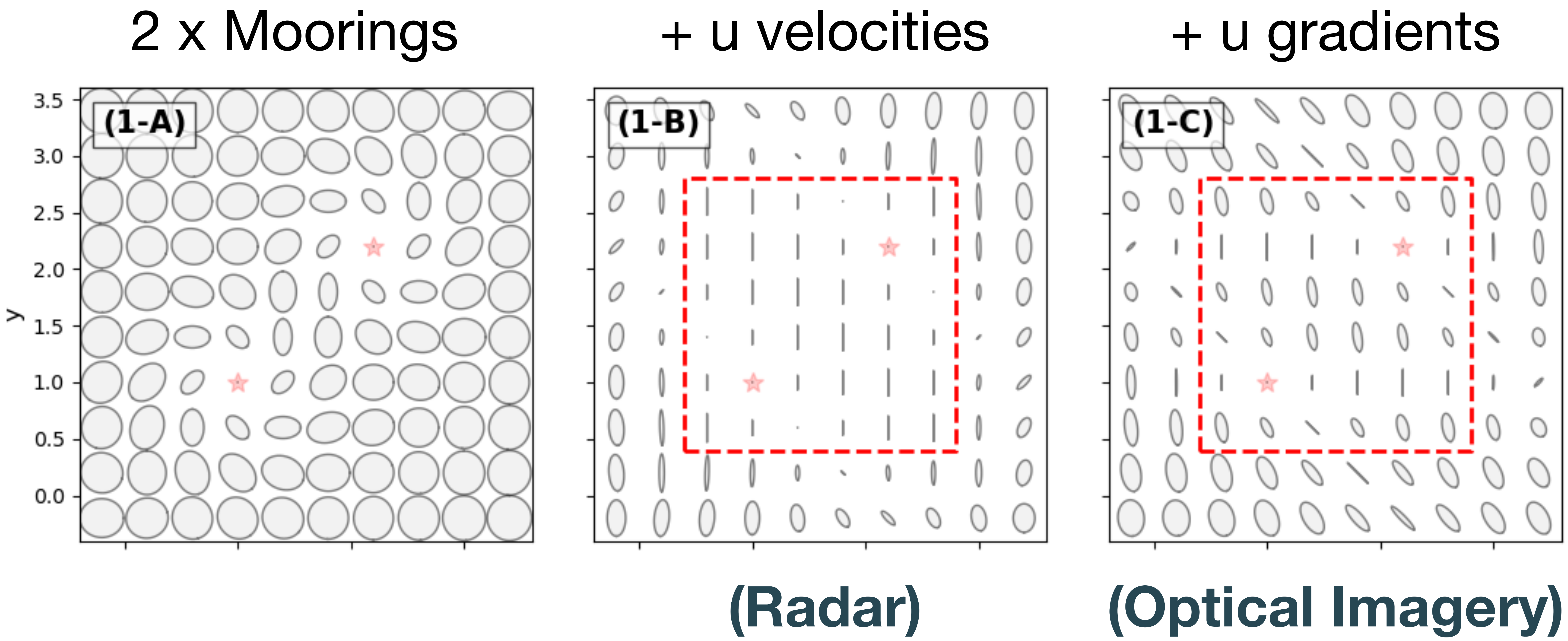
WC Edge, MD Rayson, NL Jones, AP Zulberti, LC Astfalck. Uncovering involute spatiotemporal dynamics of surface currents in the ocean. *2024 Ocean Modelling and Observations Workshop*.

Warm-up GPs

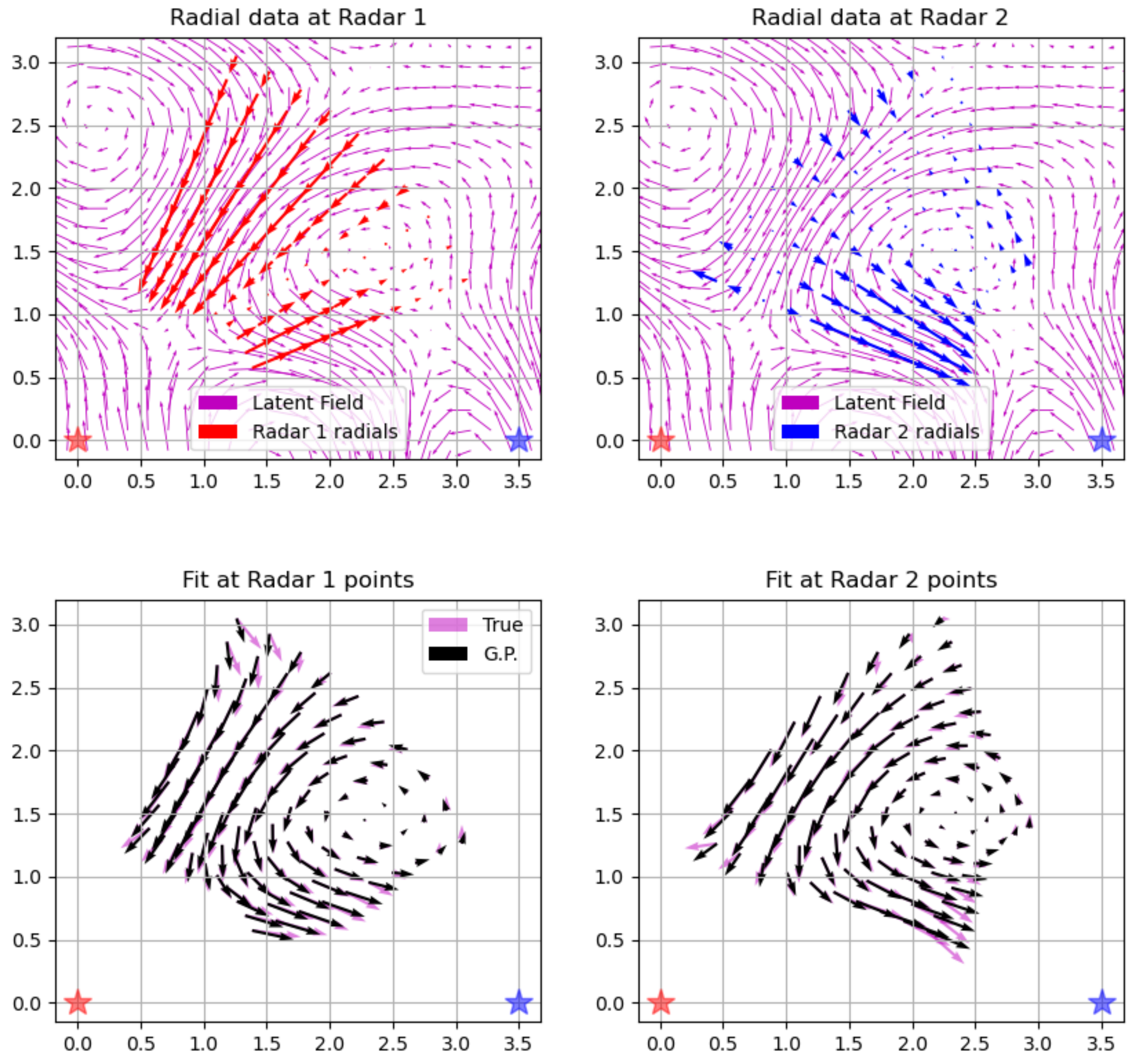


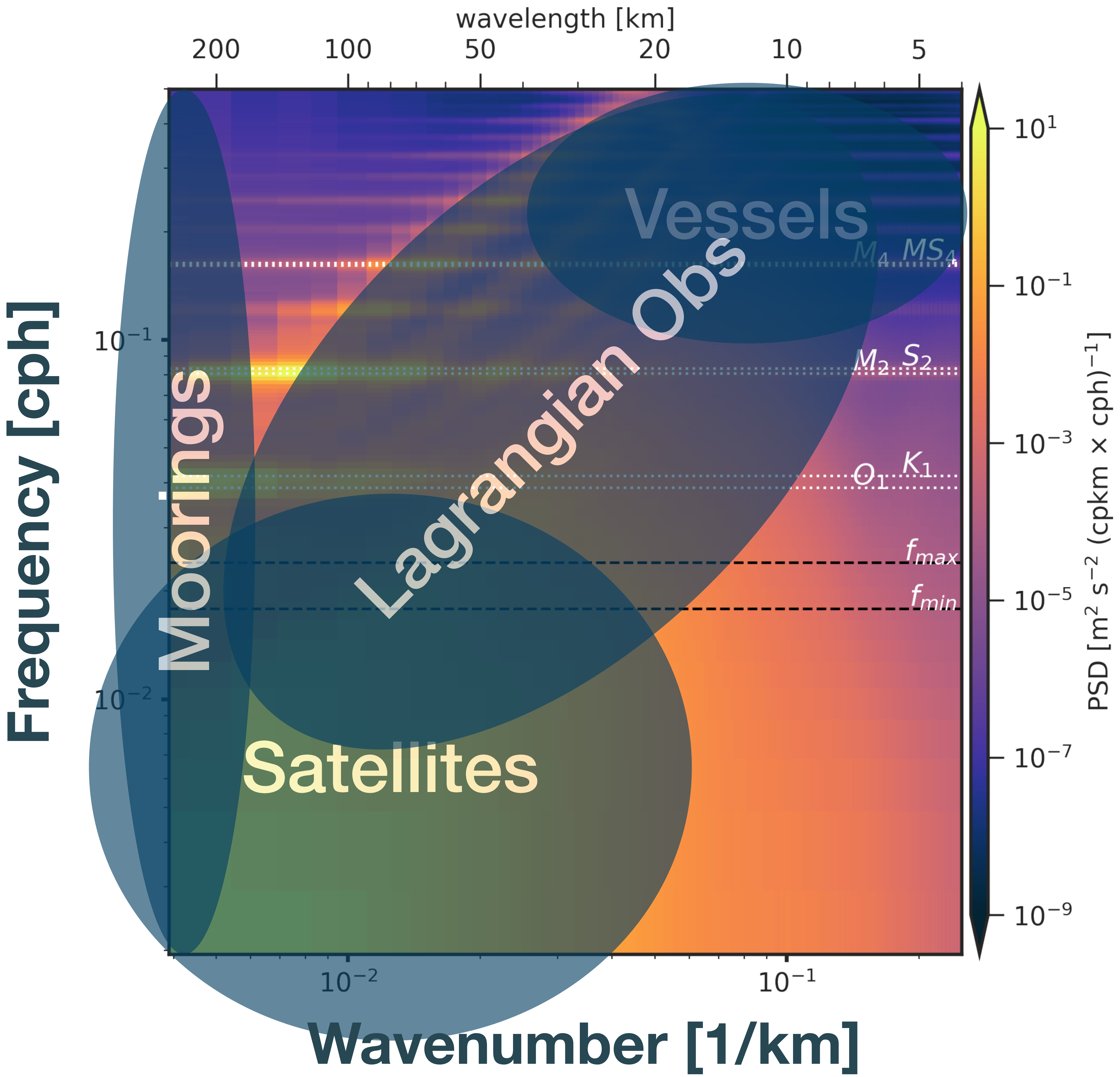
Physics Informed Covariance

Merging Data



2 x Moorings
+ u velocities
+ u gradients
(1-A)
(1-B)
(1-C)
y
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
(Radar)
(Optical Imagery)

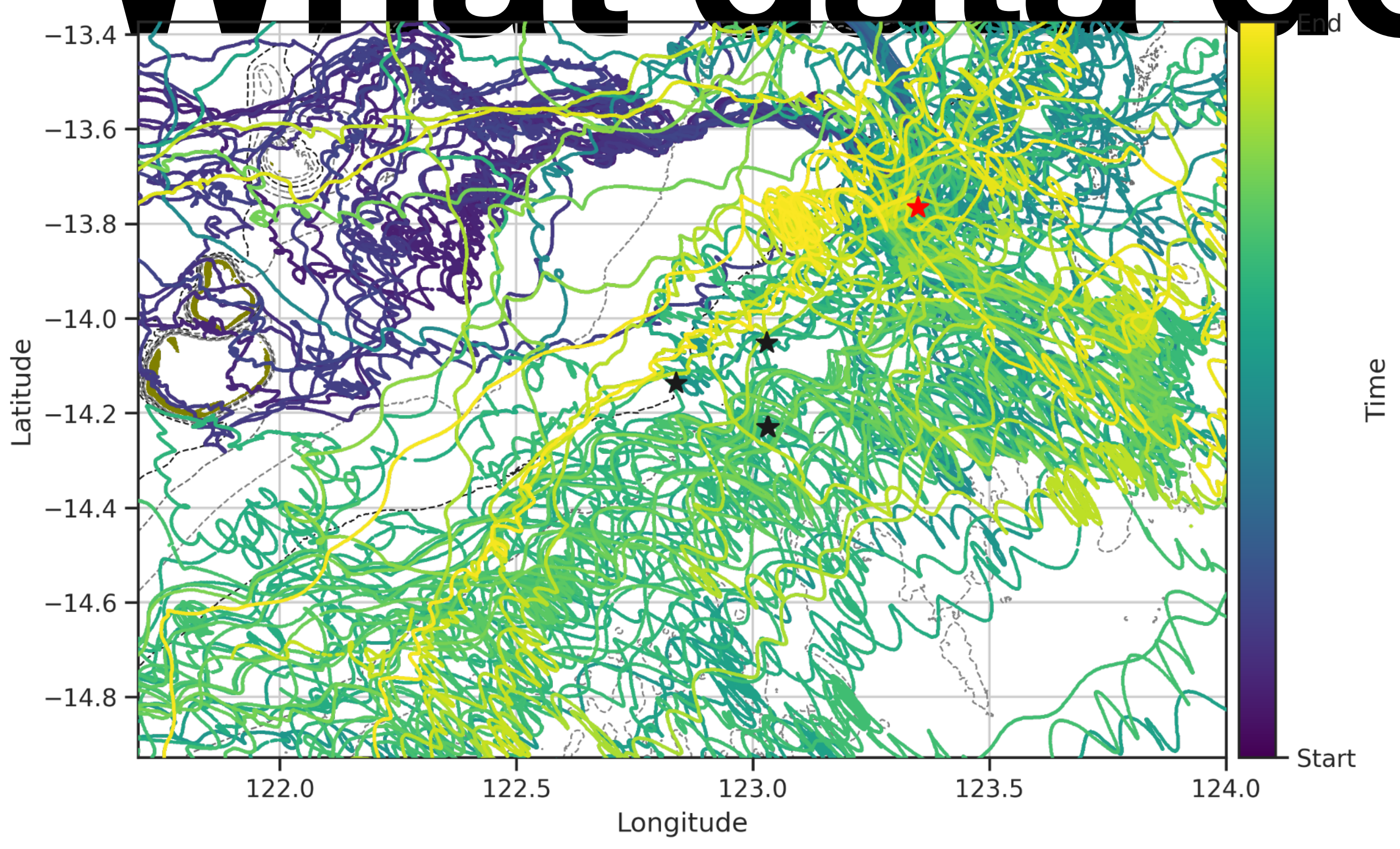

Radial data at Radar 1
Latent Field
Radar 1 radials
Radial data at Radar 2
Latent Field
Radar 2 radials
Fit at Radar 1 points
True
G.P.
Fit at Radar 2 points
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5



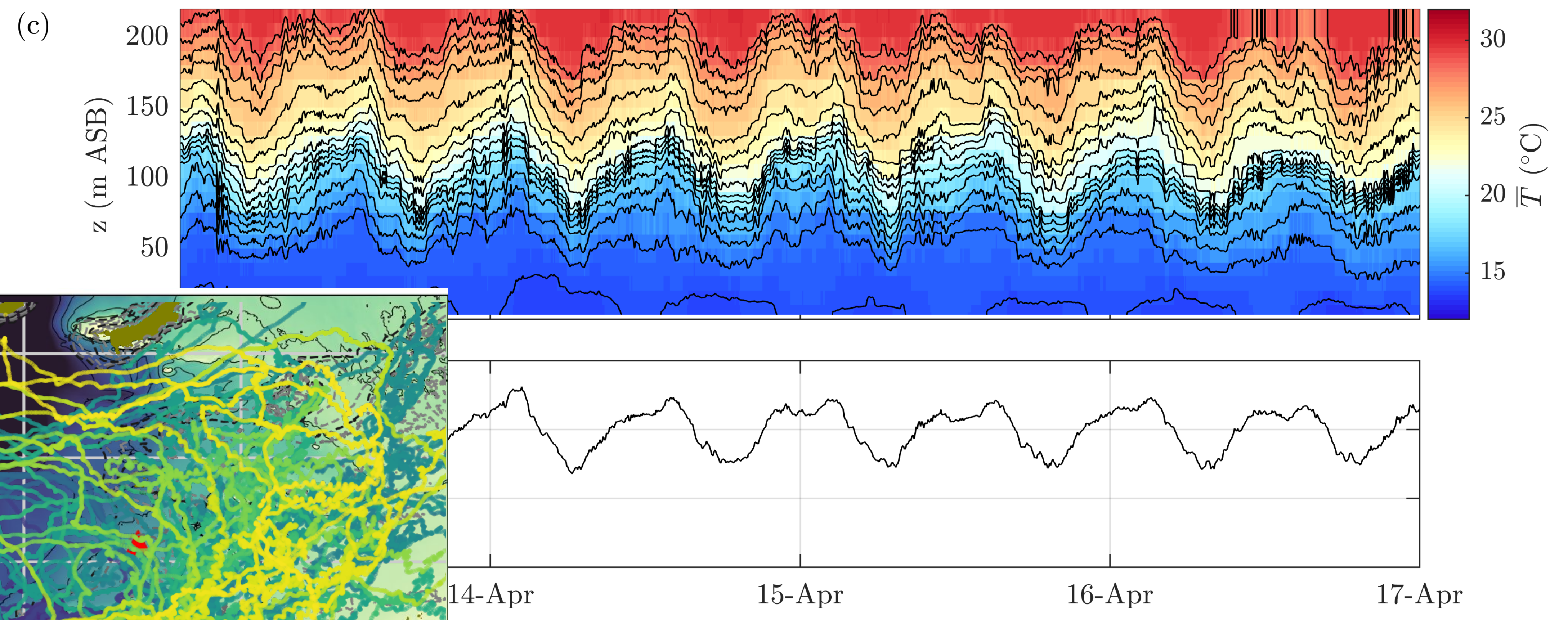
wavelength [km]
200
100
50
20
10
5
Vessels
Lagrangian Obs
Moorings
Satellites
M_4 MS_4
M_2 S_2
O_1 K_1
f_{max}
f_{min}
10^{-1}
10^{-2}
Frequency [cph]
Wavenumber [1/km]
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
PSD [m^2 s^{-2} (cpkm × cph)$^{-1}$]

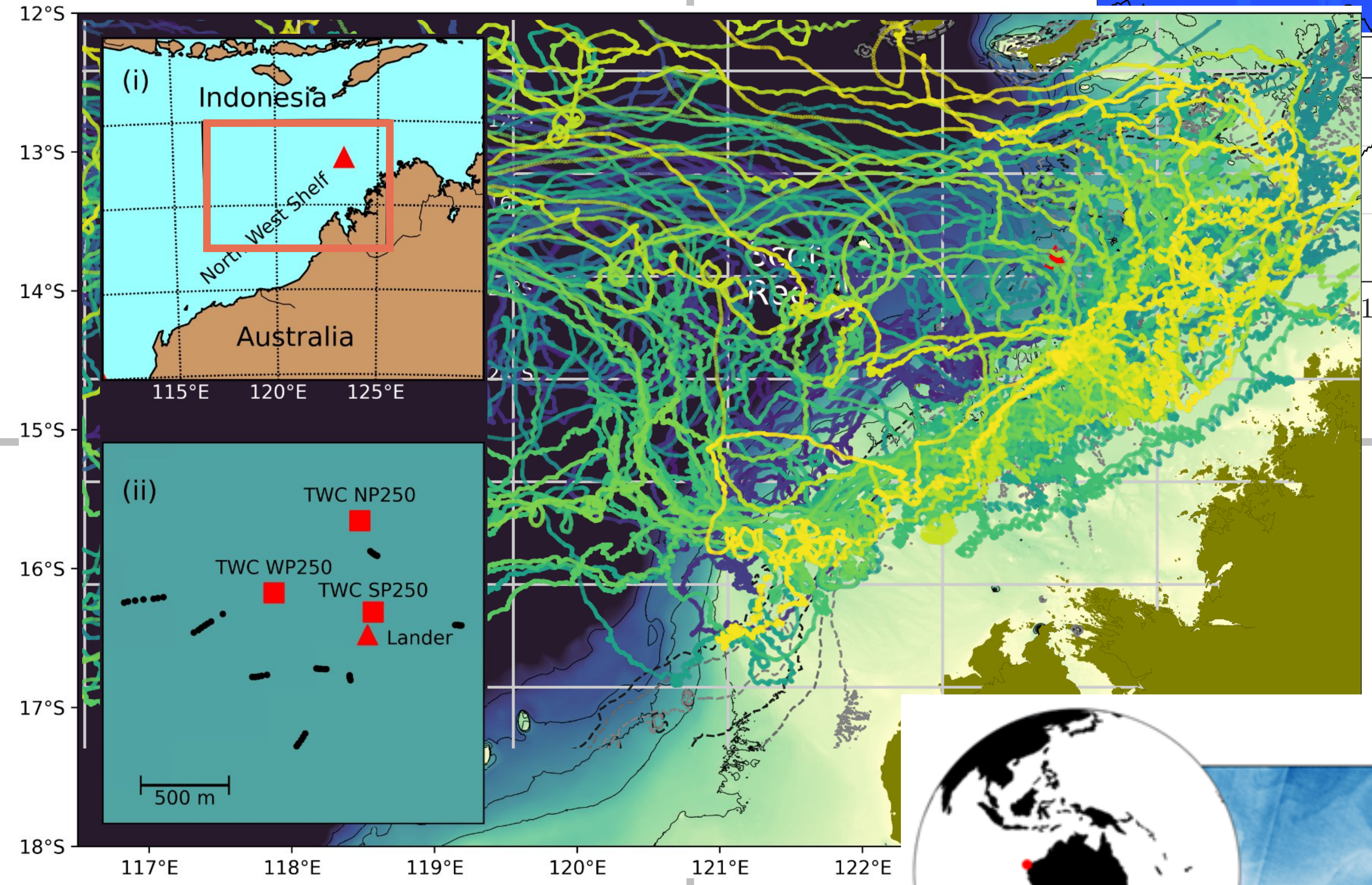
What data do we have?

Drifters

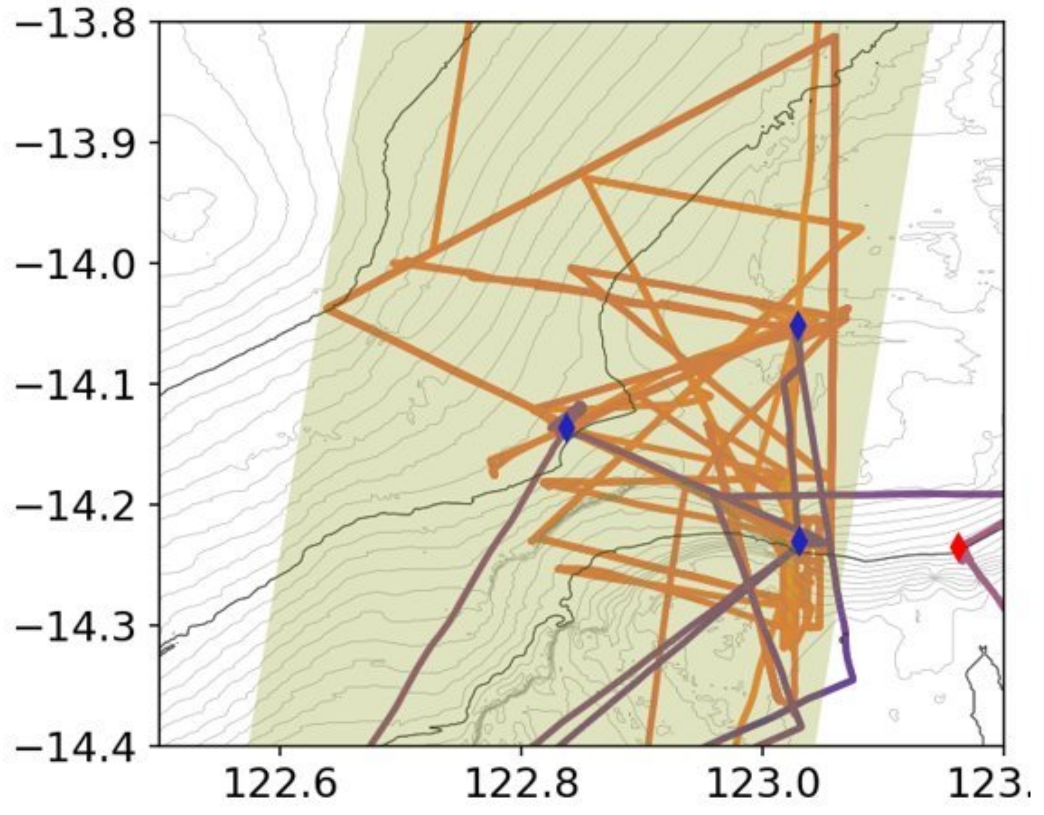


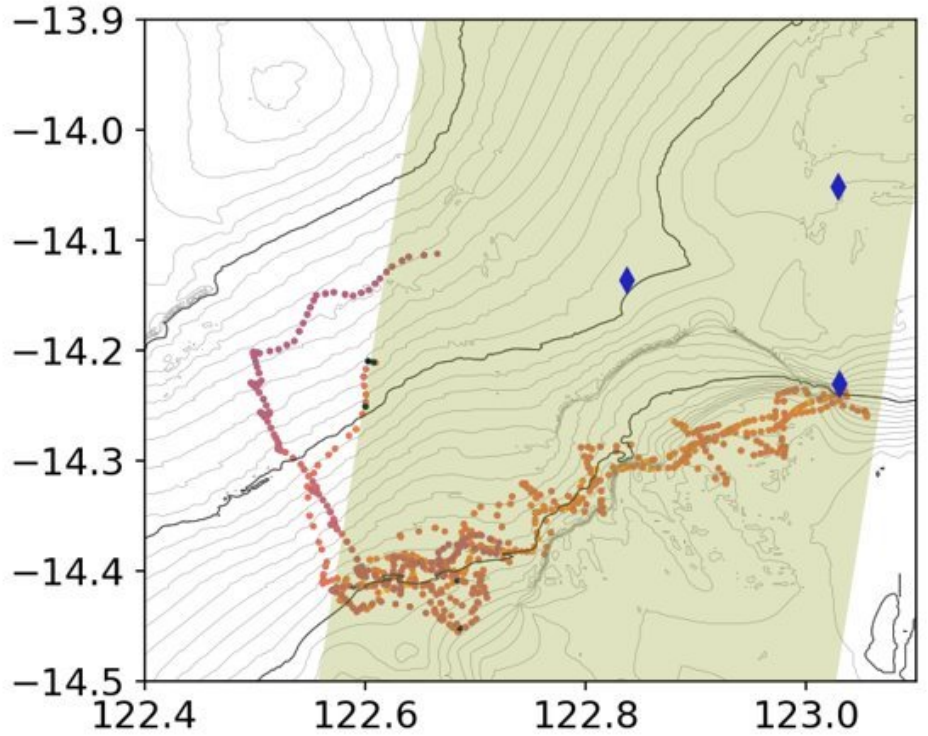
Moorings



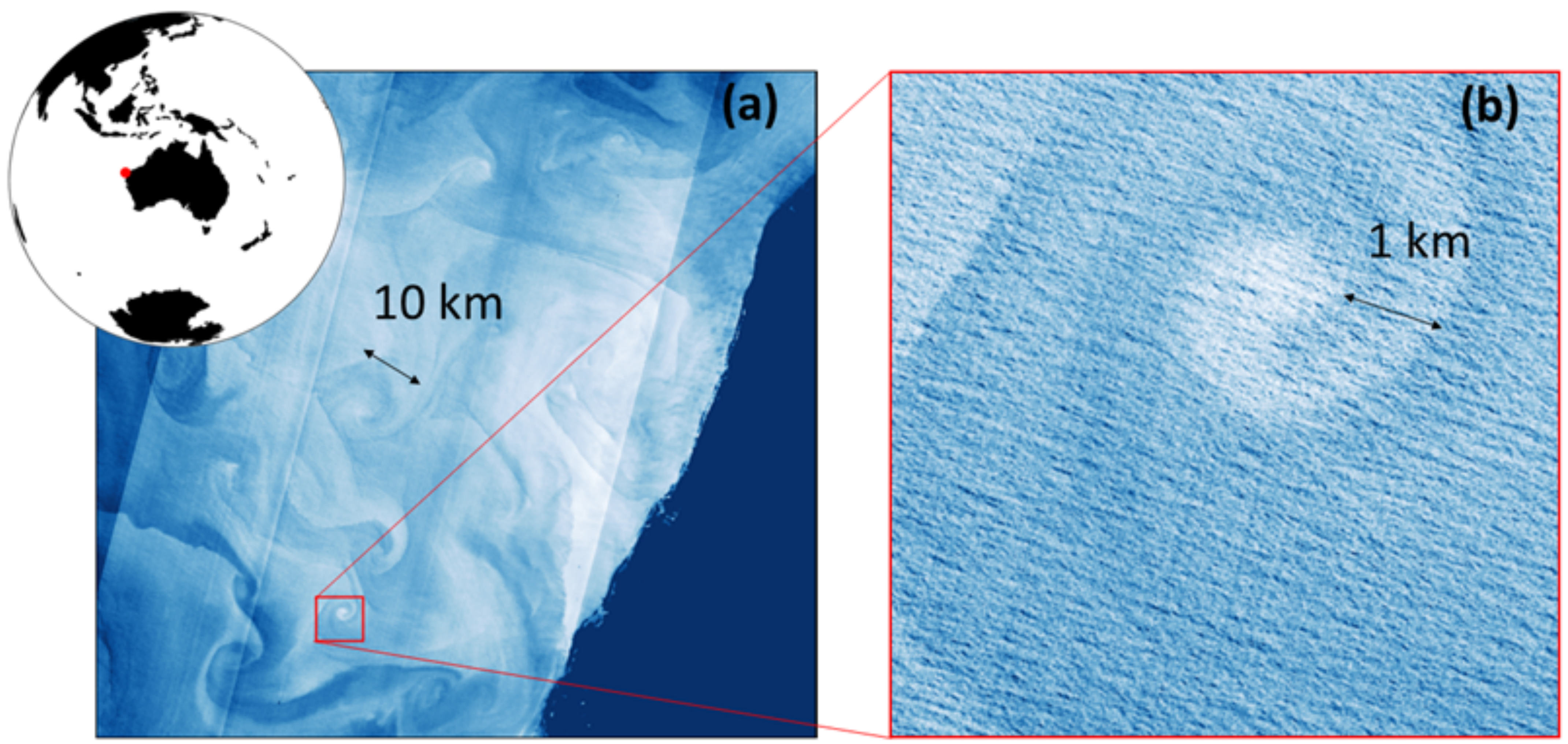


Vessel Obs

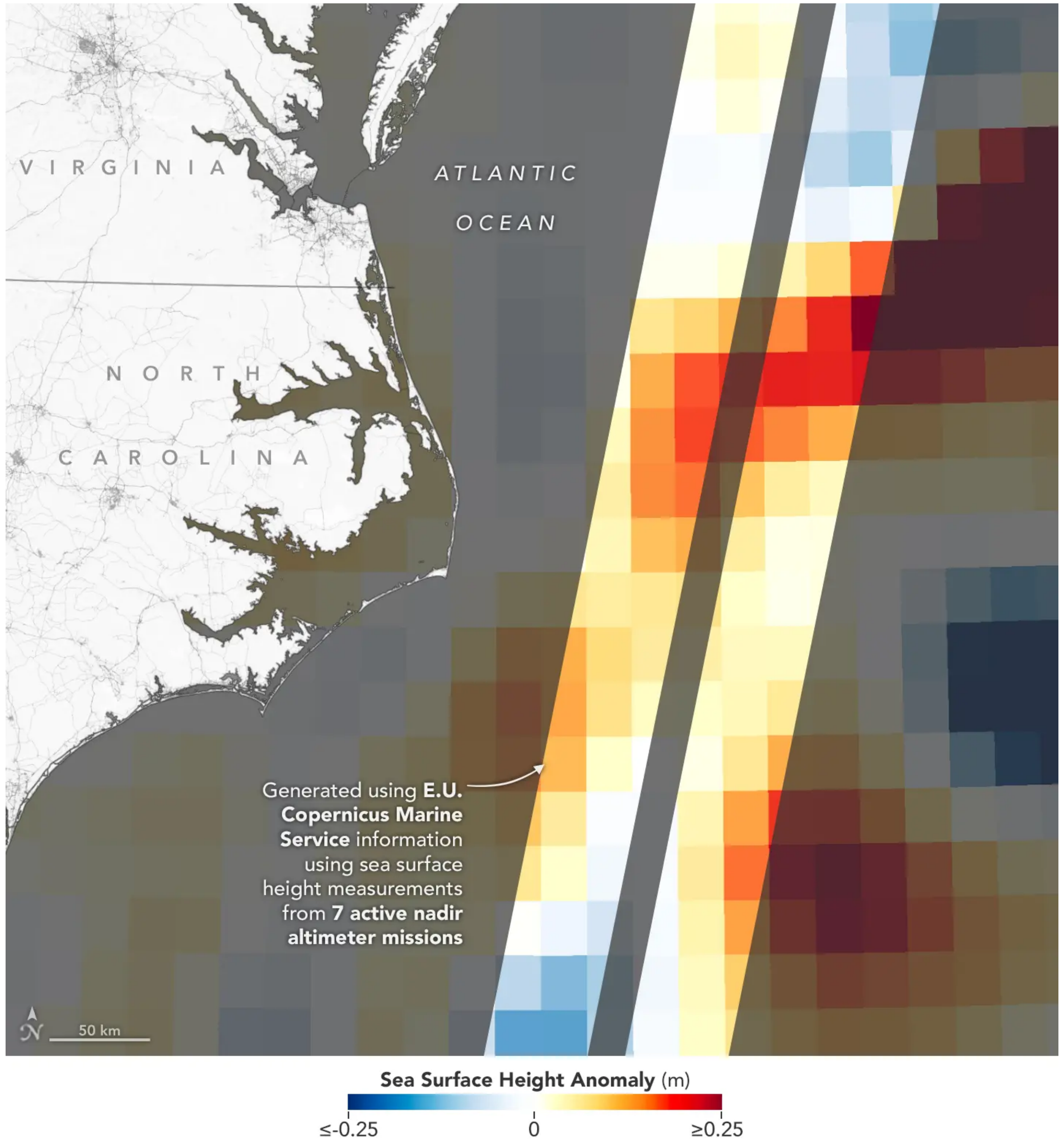


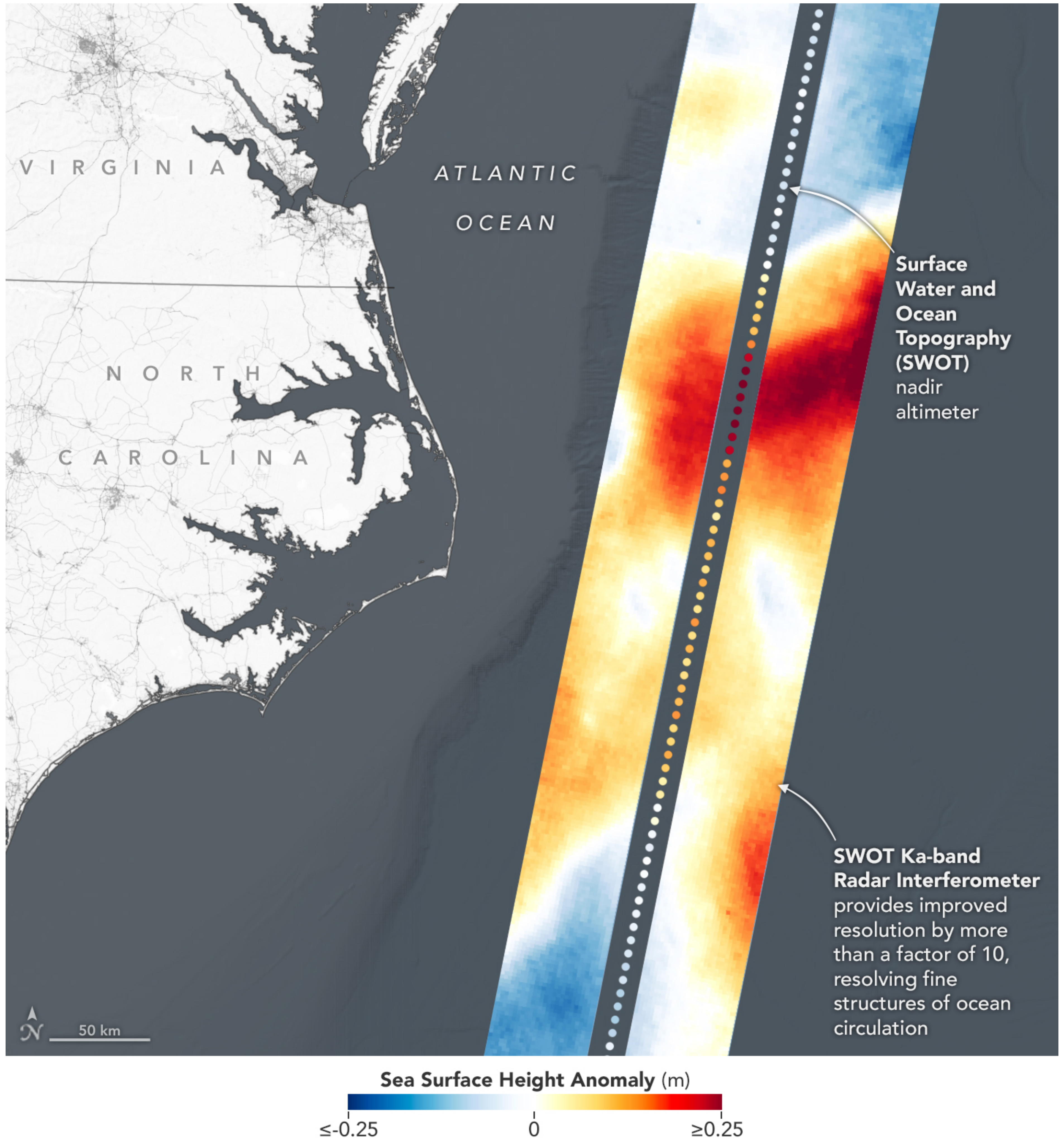


Satellite



A shout out to SWOT





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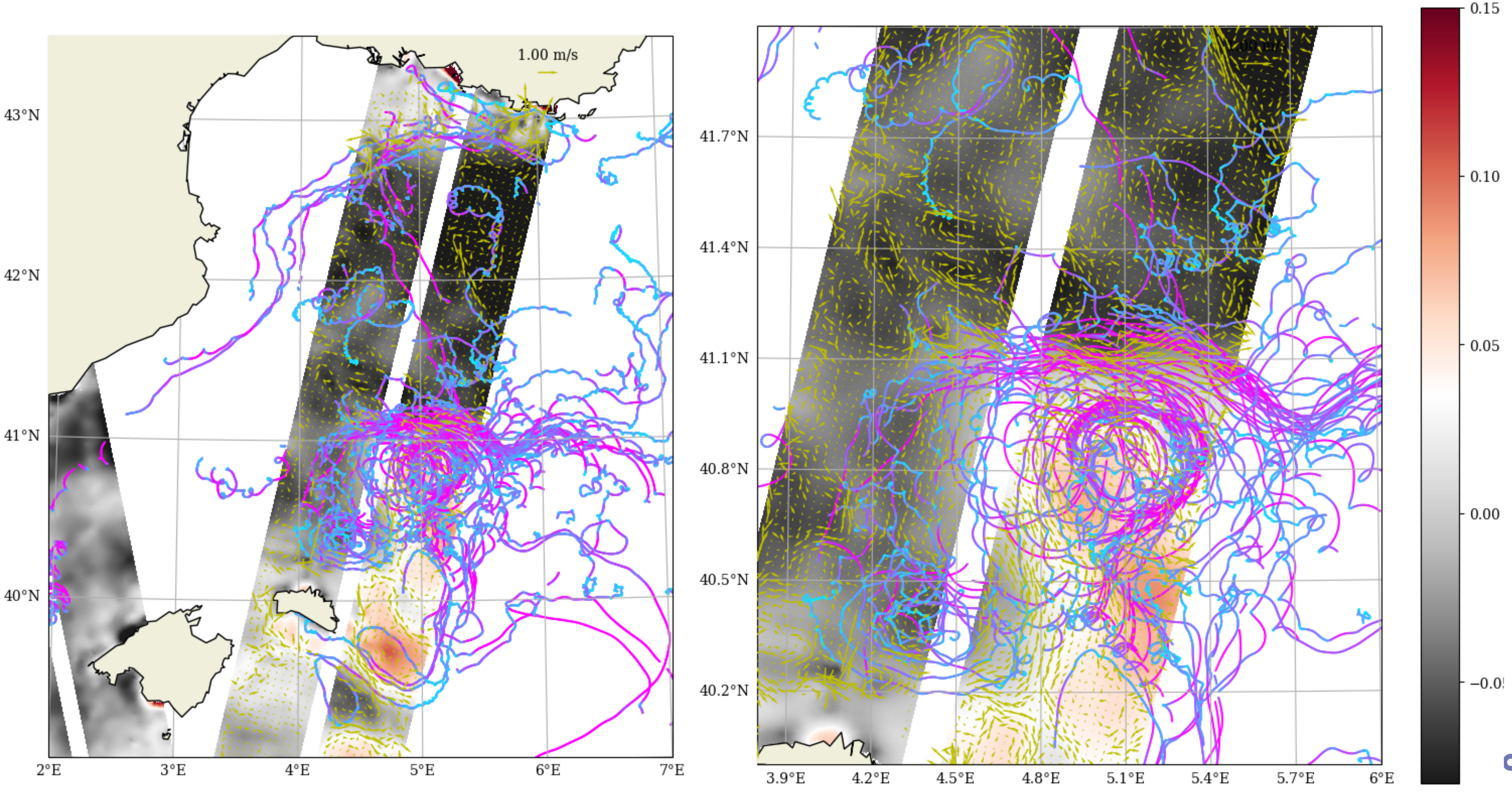

Ongoing Challenges

Computation

Kernels

Nonlinearity

From 3D to 4D

- MG Bertolacci, LC Astfalck, EJ Cripps. (2024). ‘Bayesian integration of surface current astronomic potential and stochasticity’. In preparation for Ocean Engineering.
- AP Zulberti, NL Jones, GN Ivey. (2020). Observations of enhanced sediment transport by nonlinear internal waves. *Geophysical Research Letters*
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- JM Lilly, AM Sykulski, JJ Early, & SC Olhede. (2017). Fractional Brownian motion, the Matérn process, and stochastic modeling of turbulent dispersion. *Nonlinear Processes in Geophysics*.
- AM Sykulski, SC Olhede, JM Lilly, & E Danioux. (2016). Lagrangian time series models for ocean surface drifter trajectories. *Journal of the Royal Statistical Society Series C: Applied Statistics*.
- A Wilson, & R Adams. (2013). Gaussian process kernels for pattern discovery and extrapolation. In *International conference on machine learning* (pp. 1067-1075). PMLR.
- Ponte, A. L., Klein, P. (2015). ‘Incoherent signature of internal tides on sea level in idealized numerical simulations’. *Geophysical Research Letters*.
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- Zulberti ACOMO

astfalckl.github.io